MANICALAND STATE UNIVERSITY

OF

APPLIED SCIENCES

FACULTY OF ENGINEERING, APPLIED SCIENCES & TECHNOLOGY

DEPARTMENT OF APPLIED STATISTICS

MODULE: TIME SERIES ANALYSIS

CODE: ASTA 222

SESSIONAL EXAMINATIONS

APRIL 2023

EXAMINER: MR A.CHAKAIPA

INSTRUCTIONS

1. Answer **All** in Section A.
2. Answer **three** questions in Section B.
3. Start a new question on a fresh page.
4. Total marks: 100.

Additional material(s)

- Statistical tables, graph paper, Non-programmable electronic scientific calculator, List of formulae.

SECTION A [40 MARKS]

Answer **ALL** questions in this section

A 1

(a) Explain in brief the Box-Jenkins Technique. (3)

(b) Differentiate between a sesonal and trend variations in time series analysis. (4)

A 2

(a) Sate the use of seasonal analysis in time series. Consider the following quarterly sales of houses by Valley Estates in Cape Peninsula for the period 2008 to 2011 (2)

Quarter	2008	2009	2010	2011
Q1	54	58	49	60
Q2	55	61	55	64
Q3	94	87	95	99
Q4	70	66	74	80

(b) Plot the time series for the quarterly house sales graphically. (4)

(c) Find the least squares trend line for quarterly house sales in Cape town . (5)

(d) Compute the quarterly seasonal indexes for each quarter. (8)

(e) Hence estimate the seasonally adjusted trend values for quarters 3 and 4 of 2012. Use the trend line equation and seasonal indexes obtained before. (4)

A 3

Let $Z_t = a_t + 0.3a_{t-1}$ where $a_t \sim NID(0, \sigma_{t^2})$ and a_t is a white noise term.

(a) Define white noise (1)

(b) Find the mean of Z_t (2)

(c) Variance of Z_t (2)

(d) The auto covariance function of Z_t (3)

(e) The autocorrelation function of Z_t (2)

SECTION B [60 MARKS]

Answer any **THREE** questions in this section

B 4

Suppose $Z_t = 10 + 3t + X_t$, where X_t is a zero mean stationery process with autocovariance function γ_k

(a) Find the mean of Z_t (3)

(b) Find the autocovariance function of Z_t (3)

(c) Is Z_t stationary? (why or why not?) (2)

Suppose that an Autoregressive process of order 2, AR(2) process is given by $Z_t = \frac{1}{3}Z_{t-1} + \frac{2}{9}Z_{t-2} + a_t$

(d) How do you check for stationarity of an AR process. Hence show that Z_t is a stationary process. (3)

(e) Showing all your working, deduce that the autocorrelation function (acf) of Z_t is given by $\rho_t = \frac{16}{21}\left(\frac{2}{3}\right)^{|k|} + \frac{5}{21}\left(\frac{-1}{3}\right)^{|k|}$ for $k = 0, 1, 2, ...$ (9)

B 5

(a) State and explain in brief three transformations that can be used to make a time series stationary. Suppose we have a process given by $Z_t = 5 + 2t + a_t$ where a_t is white noise. (6)

(b) Show that X_t is not stationary. (3)

(c) Verify that the process is now stationary if we difference once. (4)

Suppose Z_t is stationary with auto-covariance function γ_k

(d) Show that $W_t = \bigtriangledown Z_t = Z_t - Z_{t-1}$ is stationary (3)

(e) Show that $U_t = \bigtriangledown Z_t^2 = \bigtriangledown(Z_t - Z_{t-1})$ is stationary. (4)

B 6

For an AR(p) process, $Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + ... + \phi_p Z_{t-p} + a_t$ Given that the Yule-Walker equation for a stationary AR(p) model is given by $\rho_k = \phi_1\rho_{k-1} + \phi_2\rho_{k-2} + ... + \phi_p\rho_{k-p}$ for $k > 1$ where $\rho_k = corr(Z_t, Z_{t-1})$ = ACF at lag k

(a) Derive the estimates of $\phi = [\phi_1, \phi_2 ..., \phi_p]$ using the Yule-Walker equations. Hence or otherwise, find the method of Moments estimates of ϕ_1 and ϕ_2 for an AR(2) process given by $Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + a_t$ (10)

There are several methods employed to test for model adequacy in time series , which include the following

(b) Test for independence (5)

(c) Test for distribution of residuals For each of the above tests, specify any two tests employed, any expected results (including deviations) and hypotheses where necessary. (5)

B 7

Consider an AR(1) process given by $Z_t = \phi Z_{t-1} + a_t$ Using the least squares method show that $\hat{\phi} = \frac{\sum_{t=2}^{n} Z_t Z_{t-1}}{\sum_{t=2}^{n} (Z_{t-1})^2}$

(a) Sate an underlying assumptions to be met for the derivation above Show that the Maximum Likelihood of $\hat{\phi} \approx r_1$ (9)

(b) State any theorem or lemma needed to arrive at derivation above. (11)

END OF QUESTION PAPER