MANICALAND STATE UNIVERSITY OF APPLIED SCIENCES

FACULTY OF APPLIED SCIENCES & TECHNOLOGY

DEPARTMENT OF APPLIED STATISTICS

MODULE: STOCHASTIC PROCESSES

CODE: HAST 428

SESSIONAL EXAMINATIONS

JUNE 2023

DURATION: 3 HOURS

EXAMINER: D. MHINI

INSTRUCTIONS

1. *Answer **All** in Section A*
2. *Answer **three** questions in Section B.*
3. *Start a new question on a fresh page*
4. *Total marks 100*

Additional material(s): *Non-programmable electronic scientific calculator.*

SECTION A (*Answer **ALL** Questions from this Section*)[40]

A1. (a) Define Brownian motion process and state its properties. **[5]**

(b) Show that $E[(B_t - B_s)^2] = t - s$. **[5]**

A2. (a) Define a martingale and state its properties. **[3]**

(b) Prove that $X_t = B_t + 4t$ is a martingale. **[5]**

A3.(a) Define the following:

(i) Stochastic processes. **[2]**
(ii) State of a Markov chain. **[2]**
(iii) A counting processes. **[2]**
(iv) Holding time. **[2]**

(b) (i) Explain the what you understand by a queuing system described in Kendall-Lee notation as $M/M/2$. **[3]**

A4. Consider the Markov chain with three states, $S = \{1,2,3\}$, that has the following transition matrix.

$$P = \begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.4 & 0 & 0.6 \\ 0 & 0.8 & 0.2 \end{pmatrix}$$

(a) Draw the state transition diagram for this chain. **[6]**
(b) Find $P(X_2 = 3 | X_0 = 1)$. **[5]**

SECTION B (*Answer any **THREE** Questions from this Section*)**[60]**

B5. Consider the queuing model $(M/M/1):(GD/R/R)$.

(a) For the process above, define states of the system and obtain the probability equations of queuing process. **[5]**

(b) Solve the steady- state equations and prove that $p_n = \binom{R}{n} n!\, \rho^n \rho_0, n = 1,2, \ldots R.$ **[5]**

(c) Find the value of p_o. **[5]**

(d) Prove that $L_s = R + \frac{1}{\rho}(1 - p_0)$. **[5]**

B6. (a) Let $X \sim Poison(\lambda)$. Find

(i) PGF of X. **[4]**

(ii) Use the PGF to find the mean and variance of X. **[6]**

(b)(i) State the Markov property. **[5]**

(ii) State conditions that a counting processes must satisfy. **[5]**

B7. (a) Let $t \geq s$ then compute $E[B_s^2 B_t - B_s^3]$. **[8]**

(b)Prove that $M_t = B_t^2 - t$ is a martingale with respect to the filtration F_t. **[4]**

(c) State and explain any four service discipline. **[8]**

B8. (a) State and prove Chapman-Kolmogrov equation for a discrete Markov chain. **[10]**

(b) The city branch of a bank, during lunch hour, has four tellers to serve its customers. Customers are assumed to arrive according to Poison process with mean rate of five per two minutes. A customer can go to any teller who is free, but when all tellers are busy customers wait in a single waiting line. The service time at each teller can be assumed to follow an exponential distribution with a mean of 1.5 minutes.

Determine the values of L_q and explain your answer. **[10]**

END OF EXAMINATION PAPER