MANICALAND STATE UNIVERSITY OF APPLIED SCIENCES

FACULTY OF ENGINEERING, APPLIED SCIENCES & TECHNOLOGY

DEPARTMENT OF MINING & MINERAL PROCESSING ENGINEERING
DEPARTMENT OF CHEMICAL & PROCESSING ENGINEERING
DEPARTMENT OF METALLURGICAL ENGINEERING

MODULE: ENGINEERING MATHEMATICS IV

CODE: ENGT 224

SESSIONAL EXAMINATIONS
APRIL 2023

DURATION: 3 HOURS

EXAMINER: MR J. MANYEMBA

INSTRUCTIONS

1. *Answer **All** in Section A*
2. *Answer **Three** questions in Section B.*
3. *Start a new question on a fresh page*
4. *Total marks 100*

Additional material(s): *Non-programmable electronic scientific calculator, Statistical Tables.*

SECTION A [40 marks]

Answer **ALL** Questions being careful to number them A1 to A3.

A1. Define the following terms:

(a) Outlier, [2]

(b) Residuals, [2]

(c) serial correlation. [2]

(d) multiple regression, [2]

(e) coefficient of determination, and [2]

(f) simple linear regression. [2]

A2. Consider the data given in the table below.

x	8	13	18	6	30	22	32	40
y	28	37	63	24	101	80	115	156

(a) Construct a scatter plot and comment. [3]

(b) Find the least squares regression line. [5]

(c) Construct an ANOVA table and test appropriate hypotheses at the 5% level of significance. [8]

(d) Find the 95% confidence interval for the estimate of y when $x = 45$. [6]

A3. State and describe any **Three** types of residuals. [6]

SECTION B [60 marks]

Answer any **THREE** Questions being careful to number them B4 to B7.

B4. (a) Explain in detail any **two** methods that can be used to detect autocorrelation. [4]

(b) Explain any **three** effects of autocorrelation. [6]

(c) Data on daily electricity consumption was collected over a period of 17 consecutive days. There were 5 independent variables and a regression model was fitted. Part of the results of the analysis is given below:

$$n = 17, \qquad \sum_{t=1}^{17} e_t^2 = 722072.36, \qquad \sum_{t=1}^{17}(e_t - e_{t-1})^2 = 1865796.63.$$

Test for positive autocorrelation using the Durbin-Watson test. [10]

B5. (a) Let $\mathbf{Y} = \mathbf{X}\beta + \mathbf{e}$ be a multiple regression model where $\mathbf{Y}$ is the vector of dependent variables, $\mathbf{X}$ is the matrix containing the independent variables and $\mathbf{e} \sim N(\mathbf{0}, \sigma^2\mathbf{I})$ is the vector of the error terms.

(i) Show that the least squares estimator of β is $\hat{\beta} = (\mathbf{X}^t\mathbf{X})^{-1}\mathbf{X}^t\mathbf{Y}$ [5]

(ii) Show that $\mathbf{Var}\ (\hat{\beta}) = \sigma^2\ (\mathbf{X}^t\mathbf{X})^{-1}$ [5]

(b) Consider the following data. Using matrix method,

x_i	4	1	2	3	3	4
y_i	16	5	10	15	13	22

(i) Find $\mathbf{Y}^t\mathbf{Y}$, $\mathbf{X}^t\mathbf{X}$ and $\mathbf{X}^t\mathbf{Y}$ [2,2,2]

(ii) Find the least squares estimate of the model $\mathbf{Y} = \mathbf{X}\beta + \mathbf{e}$ [4]

B6. (a) Define multicollinearity. [2]

(b) Give and briefly explain **two** methods of removing/reducing multicollinearity. [5]

(c) Briefly discuss the relative merits of backward elimination as used in multiple linear regression. [5]

(d) Show that the Total Sum of Squares (SST) can be partitioned into Regression Sum of Squares (SSR) and Residual Sum of Squares (SSE). [4]

(e) Show that the Residual Sum of Squares (SSE) can be partitioned into Pure Error Sum of Squares (SSPE) and Lack of Fit Sum of Squares (SSLF). [4]

B7. (a) Briefly explain the forward selection procedure as used in multiple linear regression. [6]

(b) Explain why a researcher would prefer the stepwise method over forward selection method in regression model building. [4]

(c) Given a simple linear regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$ where β_0 and β_1 are constants, ϵ_i represent the error term which is assumed to follow the $N(0, \sigma^2)$ distribution.

(i) State the least squares estimators of β_0 and β_1. [3]

(ii) Show that variance of $\hat{\beta}_1 = \frac{\sigma^2}{S_{xx}}$. [3]

(iii) Show that variance of $\hat{\beta}_0 = \sigma^2[\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}]$. [4]

END OF EXAMINATION PAPER