MANICALAND STATE UNIVERSITY OF APPLIED SCIENCES

FACULTY OF ENGINEERING

DEPARTMENT OF MINING & MINERAL PROCESSING ENGINEERING
DEPARTMENT OF CHEMICAL & PROCESSING ENGINEERING
DEPARTMENT OF METALLUGICAL ENGINEERING

MODULE: ENGINEERING MATHEMATICS III

CODE: ENGT 214

SESSIONAL EXAMINATIONS
June 2023

DURATION: 3 HOURS

EXAMINER: D. MHINI

INSTRUCTIONS

1. *Answer **All** in Section A*
2. *Answer **three** questions in Section B.*
3. *Start a new question on a fresh page*
4. *Total marks 100*

Additional material(s): *Non-programmable electronic scientific calculator.*

SECTION A: Answer ALL questions in this section [40]

A1. (a) Derive the central difference formula for second order derivatives and the associated error. **[7]**

(b) Given that $f(x) = xe^x$ use the central difference formula with $h = 0.1; 0.01; 0.0001$ and 0.0001 to find the approximations to $f''(0.5)$. Compare the calculated value with the true value $f''(0.5) \approx 4.121803177$. **[7]**

A2.(a) Solve $y' = y - x; y(0) = 2$ using Runge Kutta order 1 method with $h = \frac{1}{4}; i = 0,1,2,3$ successively. **[10]**

(b) Compare and contrast the single-step methods and multi-step methods explaining how the methods work. **[5]**

(c) Define the following terms

(i) backward error analysis **[2]**

(ii) truncation error **[2]**

(iii) absolute error **[2]**

A3. (a) Define the interval of absolute convergence of RK methods for the test problem $y' = f(x, y); y(x_0) = y_0$. **[3]**

(b) Discuss the method of guaranteeing accuracy in the solution of an initial value problem using RK methods**.** **[2]**

SECTION B: Answer ANY THREE questions in this section. [60]

B4. (a) Derive composite Trapezium's rule:

$\int_a^b f(x)dx \approx \frac{1}{2}h[f_0 + 2f_1 + 2f_2 + \cdots + 2f_{n-1} + f_n]$ and show that the associated truncation error is $-\frac{1}{12}\frac{b-a}{h^2}f^{(2)}(z)$. **[14]**

(b) Consider $f(x) = 2 + \sin(2\sqrt{x})$

(i) Show that the exact value of the definite integral $\int_1^6 2 + \sin(2\sqrt{x})dx$ is 8.183479. **[2]**

(ii) Investigate the error when the composite trapezoidal rule is used over[1,6] with $h = 0.5$ **[4]**

B5. (a) Use Jacobi iterative method to solve the system of equation to 4 decimal place for $i = 0; 1; ... 5$.

$$5x + y + z = 10$$

$$x + 6y - 2z = 7$$

$$x - 3y + 7z = 16$$

Hence estimate the value of x, y, z to 1 s.f. **[10]**

(b) The equation $x^4 + 2x^3 - x - 1 = 0$ has a root in the interval [0,1]. Use the Bisection method to approximate the root. **[10]**

B6 (a) Use Heun method to solve the IVP

$y' = y - x; y(0) = 2$ on the interval [0; 1] with = 0.1, for $i = 0; 1; 2$. **[10]**

(b) (i) Sketch on a single diagram the graphs of $y = Cosx$ and $y = 2x$ for $0 \leq x \leq \frac{\pi}{2}$. Hence show that there is only 1real root for the equation

$Cosx = 2x$. **[2]**

(ii) Show that the root lies between $x = 0.2$ and $x = 0.6$. **[3]**

(iii) Starting with $x_0 = 0.5$, use the Newton Raphson method to find the root correct to 4 decimal places. **[5]**

B7 (a) Use the Modified Euler's method to solve $y' = y^2 + 1; y(0) = 0$ on the interval [0,1] with $h = 0.1; i = 0,1,2$. **[8]**

(b) Compute the IVP in (a) using $RK4$ and show which method is more accurate if the true value is 0.3093363. **[10]**

(c) Calculate the error on the methods used above. **[2]**

END OF EXAMINATION PAPER