MANICALAND STATE UNIVERSITY OF APPLIED SCIENCES

FACULTY OF ENGINEERING, APPLIED SCIENCES & TECHNOLOGY

DEPARTMENT OF APPLIED STATISTICS

MODULE: ORDINARY DIFFERENTIAL EQUATIONS

CODE: ASTA 214

SESSIONAL EXAMINATIONS

APRIL 2023

DURATION: 3 HOURS

EXAMINER: MR M. TSODODO

INSTRUCTIONS

1. *Answer* ***All*** *in Section A*
2. *Answer* ***two*** *questions in Section B.*
3. *Start a new question on a fresh page*
4. *Total marks 100*

Additional material(s): *Non-programmable electronic scientific calculator.*

SECTION A (40 marks)

Question 1

a) Show that the following differential equation is exact and find its general solution.

$$(y\cos x + 4x)dx + (\sin x + 2y)dy = 0$$

[5]

b) By finding an integrating factor, solve the initial value problem

$$(3x^2 + 8y)dy + 2xydx = 0, \qquad y(0) = 1$$

[5]

Question 2

Use the transformation $xy = t$ to solve the equation

$$(1 + x^2y^2)y + (xy - 1)^2xy' = 0$$

[7]

Question 3

Solve the Bernoulli equation

$$xy' + y = x^2y^2$$

[6]

Question 4

Use the method of reduction of order to find the general solution of the differential equation

$$y'' + \frac{2}{x}y' - \frac{6}{x^2}y = 0$$

If we know $y_1 = x^2$ is a solution

[6]

Question 5

Let $F(t)$ be a periodic function of period P, starting at $t = 0$. If $L\{f(t)\}$ exists, show that it is given by $\frac{\int_0^P e^{-st}}{1-e^{-sP}} F(t)dt$

[5]

Question 6

a) Define the terms ordinary and regular singular points in connection with a generic second order linear differential equation. [3]

b) State Frobenius' Theorem for solution of differential equations about a regular singular point. [3]

SECTION B (60 marks)

Question 7

a) Show that $x = 0$ is a regular singular point of the equation

$$4xy'' + 2y' + y = 0$$

and obtain its solution near $x = 0$. [20]

b) Solve the differential equation

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

Using the variation of parameters method [10]

Question 8

a) Solve the differential equation

$$\frac{dy}{dx} = \frac{y^2+2xy}{x^2}$$

[8]

b) Let $y_1(x)$ be a solution of the Ricatti equation

$$y' = a_1(x) + a_2(x)y + a_3(x)y^2$$

Show that the transformation

$$y = y_1 + \frac{1}{u(x)}$$

reduces the Ricatti equation to a linear equation in $u(x)$ of the form

$$u'(x) = -(a_2(x) + 2a_3(x)y_1)u(x) - a_3(x).$$

Hence, solve the following equation

$$y' = \frac{2cos^2x - sin^2x + y^2}{2cosx}, \quad y_1(x) = sinx$$ [12]

c) A certain radioactive material is known to decay at a rate proportional to the amount present and 0.1 percent of the original mass decayed after one week. Find an expression for the mass at time t. [10]

Question 9

a) Derive a general expression for the Laplace transform of $\frac{d^ny}{dx^n}$.

Hence solve the following Laplace transform

$$y''' + y' = e^x, \qquad y(0) = y'(0) = y''(0) = 0.$$

[15]

b)

i) Show that any Cauchy-Euler second order differential equation can be reduced to any equation with constant coefficients by means of the substitution

$$x = e^t \ .$$ [8]

ii) Hence or otherwise find the general solution of the Euler equation

$$x^2y'' - 2xy' - 4y = 0$$ [7]

END OF QUESTION PAPER