MANICALAND STATE UNIVERSITY OF APPLIED SCIENCES

FACULTY OF ENGINEERING, APPLIED SCIENCES & TECHNOLOGY

DEPARTMENT OF APPLIED STATISTICS

MODULE: LINEAR MATHEMATICS 1

CODE: ASTA 103

SESSIONAL EXAMINATIONS

JUNE 2023

DURATION: 3 HOURS

EXAMINER: DR W. GOVERE

INSTRUCTIONS

1. *Answer **All** in Section A*
2. *Answer **three** questions in Section B.*
3. *Start a new question on a fresh page*
4. *Total marks 100*

Additional material(s): *Non-programmable electronic scientific calculator, List of formulae.*

SECTION A (*Answer ALL Questions from this Section)*

A1 Represent the following complex numbers on an Argand diagram:

a) $\frac{(3+2i)}{(2-i)}$.

b) $(3+5i)(2-2i)$.

c) $(0.5-0.3i)$ - $(0.6+1.4i)$.

[4, 3, 3]

A2. Let $M = \begin{pmatrix} 1 & 1 & -1 \\ 3 & -2 & 1 \\ 4 & 1 & -2 \end{pmatrix}$ be a given matrix

a) Evaluate $|M|$ and hence find $|3M|$

b) Evaluate M^{-1}.

c) Use the method of inverses to solve the system

$MX = (1 \quad 3 \quad 9)^t$, where $X = (x \quad y \quad z)^t$.

[3, 3, 4, 7]

A3. Find the general solution of the following differential equations.

i) $\frac{dy}{dx} = \frac{1}{1+x^2}$

ii) $\frac{dy}{dx} + 2y = e^{2x}$

[3, 4]

A4. Solve the following system of linear equations by Gauss-Jordan elimination

$$2x + y - z = -1$$
$$-2x + y + 2z = 1$$
$$x + y + z = 2$$

[6]

SECTION B (*Answer any* ***THREE*** *Questions from this Section)*

B5 a) If A= $\begin{pmatrix} 1 & 4 & -2 \\ 2 & 5 & 1 \\ -1 & 3 & -3 \end{pmatrix}$

i) Evaluate $|A|$ and hence find A^{-1}.

ii) Evaluate $A^2 - 2A$.

iii) Use the method of inverses to solve $AX = (2, 5, -1)^T$.

iv) Prove that det $(A^{-1})= \frac{1}{\det(A)}$.

b) Find the value of a for which the systems of equations have no solution.

$$3x - 2y + 3z = 4$$

$$-x + y + z = 1$$

$$2x - y + 3z = a - 2.$$

[4, 4, 4, 3, 5]

B6 a) Express the following complex numbers in polar form:

(i) $-1 - i$

(ii) $-4 - i$

b) Let $z = 10cis\frac{5\pi}{6}$ and $w = 6cis\frac{\pi}{3}$ evaluate:

(i) zw.

(ii) $\frac{z}{w}$.

c) Given the following complex numbers $z_1 = 2 - 2i$, $z_2 = \sqrt{3} + i$ and $z_3 = a + bi$ where $a \in \mathbb{R}$, $b \in \mathbb{R}$

(i) If $|z_1 z_3| = 16$, find the modulus z_3

(ii) Given further that $\arg\left(\frac{z_3}{z_2}\right) = \frac{7\pi}{12}$, determine the argument of z_3

(iii) Find the values of a and b, and hence show that $\frac{z_3}{z_1} = -2$

[2, 3, 2, 3, 4, 3, 3]

B7

a) Find the particular solution to the differential equation $\frac{dy}{dx} = xy + x$ which satisfies $y = 3$ when $x = 0$. Show your work.

b) Show that the equation $\frac{dy}{dx} = \frac{x+3y}{y-3x}$ is a homogeneous first-order equation and hence solve it.

c) Show that the equation $\cos(x+y)dx + \left(3y^2 + 2y + \cos(x+y)\right)dy = 0$ is exact and hence solve the equation.

[7, 8, 5]

B8 a) Show that if $n \neq 0,1$ then the substitution $w = y^{1-n}$ reduces the Bernoulli equation $y' + p(x)y = q(x)y^n$ to a linear equation. Hence solve the equation $\cos x \frac{dy}{dx} - y \sin x = y^3 \cos^2 x$.

b) The population of bacteria grows at a rate proportional to the number of bacteria present at any time. Initially there were 400 bacteria and after 3 hours 2000 bacteria were present. Find the number of bacteria after 10 hours.

[10, 10]

END OF EXAMINATION