MANICALAND STATE UNIVERSITY OF APPLIED SCIENCES

FACULTY OF AGRIBUSINESS AND COMMERCE

Agricultural Economics and Development

MATHEMATICS FOR AGRICULTURAL ECONOMICS

CODE: AEDT 227

SESSIONAL EXAMINATIONS
JULY 2022

DURATION: 3 HOURS

EXAMINER: MR N. CHIPUNZA

INSTRUCTIONS

1. *Answer All questions in section A*
2. *Answer any Three (3) questions in section B.*
3. *Start a new question on a fresh page*
4. *Total marks 100*

Additional material(s): *Calculator*

SECTION A: [ANSWER ALL]

Question 1

Suppose that the market for tea is described by the demand and supply functions

$$D_t = 100 - 5P_t + 3P_c$$
$$S_t = -10 + 2P_c$$

And the market for coffee by

$$D_c = 120 - 8P_c + 2P_t$$
$$S_c = -20 + 5P_c$$

Where P_t is the price of tea, $P_c$is the price of coffee, $D_t \wedge S_t$ are the quantities of tea demanded and supplied respectively and $D_c \wedge S_c$are the quantities of coffee demanded and supplied respectively. Solve for the following using matrices

i. equilibrium quantity and price of tea [4]
ii. equilibrium quantity and price of coffee [4]

Question 2

The function $f(x, y) = ax + y$ is to be maximized under the constraints $x^2 + ay^2 \leq 1,\ x \geq 0, y \geq 0$, where a is a positive real parameter.

i. Use the Langrange – multiplier method to write down the F.O.C for the maximum. [8]
ii. Taking as given that this problem has a solution, find how the optimal values of x and y change if a increases by a very small number. [4]

Question 3

For a firm with Total Cost given by

$$TC = 120 + 45q - q^2 + 0.4q^3$$

And faces a demand curve given by

$$p = 240 - 20q$$

Identify its

a. AC [2]
b. FC [2]

c. VC [2]

d. AVC [2]

e. AFC [2]

f. Profit function. [2]

Question 4

Solve the system $Ax = b$, given [8]

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & -1 \\ 1 & 3 & 4 \end{pmatrix}, \qquad b = \begin{pmatrix} 5 \\ 1 \\ 6 \end{pmatrix}$$

SECTION B [ANSWER ANY THREE (3) QUESTION]

Question 5

A closed economy is described by the following system of equations that give equilibrium conditions in the goods and the money markets, the IS and the LM relationships. The goods market is described by

$$C = 15 + 0.8(Y - T)$$
$$T = -25 + 0.25Y$$
$$I = 65 - R$$
$$G = 94$$

The money market is described by

$$L = 5Y - 50R$$
$$M = 1{,}500$$

a. find the equilibrium levels of Y and R [8]

b. Suppose you have opened up the economy that allows for imports and exports. Were net exports is given by

$$X = 50 - 10 - 0.1Y$$

i. Calculate the government's budget deficit or (surplus) in the equilibrium. [6]

ii. Calculate the trade deficit the (or surplus) in the equilibrium. [6]

Question 6

a. Solve the utility maximization problem using the Lagrange method. Find the quantities demanded of the two goods? [8]

$$maxu(x,y) = x^2 + y \qquad \text{subject to} \qquad x + 4y = 200$$

b. Find $\frac{dz}{dx}$ and $\frac{dz}{dy}$, given [12]

$$z = x^3 + 3x^2y - y^3$$

$$z = \frac{xy}{x - y}$$

$$z = xe^{-xy}$$

Question 7

A. For what values of a is B symmetric [8]

$$\begin{matrix} a & a^2 - 1 & -3 \\ a + 1 & 2 & a^2 + 4 \\ -3 & 4a & -1 \end{matrix}$$

B. Find all solutions of the system of linear equations [12]

$$\begin{aligned} 2x + y - z &= 3 \\ 3x + 3y + 2z &= 7 \\ 7x + 5y &= 13 \end{aligned}$$

Question 8

A. Verify if the following matrices are idempotent

i. $\begin{bmatrix} x & -x \\ x - 1 & 1 - x \end{bmatrix} = B$ [4]

ii. $\begin{bmatrix} \frac{1}{6} & \frac{-1}{3} & \frac{1}{6} \\ \frac{-1}{3} & \frac{2}{3} & \frac{-1}{3} \\ \frac{1}{6} & \frac{-1}{3} & \frac{1}{6} \end{bmatrix} = A$ [8]

B. Find the unknown matrix X from the equation. [4]

$$\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} X = \begin{pmatrix} 4 & -6 \\ 2 & 1 \end{pmatrix}$$

Question 9

A. Use the Cramer's Rule to solve the following systems of equations for x and y. Test the answers by substitution. [12]

(a) $3x + y = 8$
$x - 2y = 5$

(b) $x + 3y = 1$
$3x - 2y = 14$

(c) $ax - by = 1$
$bx + ay = 2$

B. Find the eigen values and corresponding eigenvectors of the following matrix [8]

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

END OF EXAMINATION