MANICALAND STATE UNIVERSITY OF APPLIED SCIENCES

FACULTY OF APPLIED SCIENCES & TECHNOLOGY

DEPARTMENT OF APPLIED STATISTICS

MODULE: ESTIMATION TECHNIQUES

CODE: ASTA 223

SESSIONAL EXAMINATIONS
DECEMBER 2023

DURATION: 3 HOURS

EXAMINER: MR M. TSODODO

INSTRUCTIONS

1. *Answer* ***All*** *in Section A*
2. *Answer* ***two*** *questions in Section B.*
3. *Start a new question on a fresh page*
4. *Total marks 100*

Additional material(s): *Non-programmable electronic scientific calculator.*
Statistical tables

SECTION A

Question 1

Identify and name any;

i) Four probability densities where the parametric space $p = 1$. [4]

ii) Two probability functions where the parametric space $p = 2$. [2]

Question 2

Let $X_1, \dots \dots . . X_n$ be a random sample obtained from a population with mean μ. Show that $\hat{\mu} = \bar{x}$ is an unbiased estimator of μ. [5]

Question 3

Consider the sample $X_1, \dots \dots . . , X_n$ drawn from a normal population with mean μ and variance δ^2. What is the MLE of:

a) $\tau = \frac{\mu}{n}$ [1]

b) The variance δ^2 [2]

c) $Log\ \sigma$. [2]

Question 4

Let Y be a random variable with mean $\mu(\alpha, \beta) = \alpha + \beta x_i$. Find the least squares estimates of α and β based on the realization $(y_1, x_1), \dots (y_n, x_n)$. [8]

Question 5

A laboratory test is 95% effective in detecting a certain disease when it is in fact, present. However, the test yields a ‘false positive’ result of 1% of the health persons tested. Suppose that 0.5% of the population has the disease. Calculate the probability that a person has the disease given that the test result is positive. [5]

Question 6

Let $X_1, \dots \dots . . X_n$ be a random sample obtained from a Bernoulli distribution that has the parameter θ.

a) Find a lower bound of the variance of T if T is an unbiased estimator of $\tau(\theta) = \theta$. [8]

b) Show that $\bar{X}$ is an UMVUE of θ. [3]

SECTION B

Question 7

Let $X_1, \dots \dots . . X_n$ be a random sample obtained from a Raleigh distribution whose density is:

$$f_X(x, v) = \frac{x}{v} \exp[{}^{-x^2}/_{2v}],\ for\ x > 0$$

a) Is the maximum likelihood estimator of V an unbiased estimator? [4]

b) Determine whether $f_X(x, v)$ is a member of the exponential class of distribution [3]
c) Find a minimal sufficient statistic V [5]
d) Find the Cramer-Rao lower bound of the variance of unbiased estimator of V. [6]
e) Find a complete statistic [6]
f) Find an UMVUE of V. [6]

Question 8

a) Let $X_1, \dots \dots \dots .. X_n$ be a random sample from a Poisson distribution with unkown parameter θ, that is. $P(X = x) = f_X(x; \lambda) = \frac{e^{-\theta}\theta^x}{x!} . I_{\{0,1,\dots\}}(x)$ where $\lambda \geq 0$.
   i) Assuming a uniform prior density θ, find the posterior distribution of θ. [10]
   ii) Assuming a gamma prior density for θ,
   Find the posterior distribution of θ. [10]
   Find the Bayes' estimator of θ [5]
   Find the posterior Bayes' estimator of $\tau(\theta) = P[X_i = 0]$ [5]

Question 9

a) Suppose a random sample $X_1, \dots \dots \dots .. X_n$ of size 2 is drawn from a Bernoulli distribution

$$P(x) = P^x(1-P)^{1-x}, \qquad x = 0,1$$

i) List all the possible samples of size 2 that can be selected [4]
ii) Find in terms of P the probability of selecting the sample $\{1,0\}$. [4]
iii) Find the likelihood function of $\{1,0\}$. Hence find the value of P which maximises the probability of selecting $\{1,0\}$. [6]
iv) Find in terms of x_1, x_2, the value of the parameter P that maximises the likelihood function $L(P)$. [8]

b) State the (Rao Blackwell Theorem). [3]
c) Suppose X is a binomially distributed random variable, ie. $X \sim Binomial(n, p)$. Show that the distribution of X is a member of the exponential class. [5]