MANICALAND STATE UNIVERSITY OF APPLIED SCIENCES

FACULTY OF APPLIED SCIENCES & TECHNOLOGY

DEPARTMENT OF APPLIED STATISTICS

MODULE: MATHEMATICAL DISCOURSE AND STRUCTURES

CODE: ASTA 123

SESSIONAL EXAMINATIONS

DECEMBER 2023

DURATION: 3 HOURS

EXAMINER: D. MHINI

INSTRUCTIONS

1. *Answer **All** in Section A*
2. *Answer **three** questions in Section B.*
3. *Start a new question on a fresh page*
4. *Total marks 100*

Additional material(s): *Non-programmable electronic scientific calculator and statistical tables.*

SECTION A (*Answer* ***ALL*** *Questions from this Section)*

A1 Define the following structures

(a) Boolean Algebra **[6]**

(b) A group **[4]**

(c) Binary operation **[1]**

A2. (a) $B = \{1, 0\}$ and let two operations $+$ and $*$ be defined on B as follows:

+	1	0
1	1	1
1	1	0

and

*	1	0
1	1	1
0	1	0

Show that B is a Boolean algebra. **[5]**

(b) Prove the absorption laws:

(i) $A \cup (A \cap B) = A$ **[3]**

(ii) $A \cap (A \cup B) = A$ **[3]**

A3. (a) Draw the parallel combination switch of $P \wedge Q$**.** **[2]**

(b) Differentiate bijective and injective mapping. **[1]**

(c) The mapping $f: \mathbb{Z} \longrightarrow \mathbb{Z}$ is given by $f(x) = x^4 - 1$.

(i) What is the image of f. **[2]**

(ii) Is f injective , surjective ? **[2]**

A4. (a) xRy means $x + y$ is an integer where $x, y \in \mathbb{Z}$. Determine whether the relation in $\mathbb{Z}$ an equivalence relation. **[5]**

(b) Solve $2x - 1 = 0(mod 15)$ **[3]**

(c) Let A and B be non- empty sets. Prove that if $A \times B = B \times A$ then $A = B$ **[3]**

SECTION B (*Answer any* ***THREE*** *Questions from this Section)*

B5 Let $A = \{a, b\}$ be a set with multiplication defined by the group

	a	b
a	a	b
b	b	b

Show that A is not a group. **[5]**

(b) Prove that $a * a = a$ **[5]**

(c) Draw a Cayley table of $(\mathbb{Z}, +)$ where $\mathbb{Z}_6$ denotes integers modulo 6. **[5]**

(d) Prove that $(\mathbb{Z}, +)$ is a group. **[5]**

B6 (a) (i) Define a function**.** **[1]**

(ii) State the principle of duality. **[1]**

(b) Prove that $p \longrightarrow (q \wedge r) \equiv (p \longrightarrow q) \wedge (p \longrightarrow r)$ is logically equivalent. **[5]**

(c) Show that the set of integers form a group under addition. **[5]**

(d) Let the function $f: \mathbb{R} \longrightarrow \mathbb{R}$ be defined by;

$$f(z) = \begin{cases} 3z - 1 & if\ z > 3 \\ z^2 + y & if\ 2 \leq z \leq 3 \\ 2z + 3 & if\ z < -2 \end{cases}$$

Find (i) $f(2)$ **[1]**

(ii) $f(4)$ **[1]**

(iii) $f(-1)$ **[1]**

(iv) $f(-3)$ **[1]**

(e)Suppose $f: A \longrightarrow B$ and $g: B \longrightarrow C$ are functions

(i) Show that if $g * f$ is injective the f is injective. **[2]**

(ii) Show that if $g * f$ is surjective then g is surjective. **[2]**

B7 (a) Solve the system of congruence

$2x \equiv 3(mod\ 5)$; $3x \equiv 2(mod\ 4)$ **[7]**

(b) Let (G, o) and $(H, \times)$ be groups. Prove that $(G \times H, \triangle)$the direct product of G and H is abelian iff G and H are both abelian. **[8]**

(c) Let $X = \{p, q, r\}$. List the elements of $\mathcal{P}\{X\}$. **[5]**

B8 (a) Prove that $(A_1 - A_2) \cap (A_1 - A_3) = A_1 - (A_2 \cup A_3)$ where A_1, A_2 and A_3 are any sets. **[5]**

(b) Define a relation. **[1]**

(c) Let $\theta: G \longrightarrow K$ be a group homomorphism. Prove that $Ker\ \theta \lhd G$. **[6]**

(d) Prove DeMorgan's laws

$(A \cup B)' = A' \cap B'$; $(A \cap B)' = A' \cup B'$ **[6]**

(e)Show that the composition of surjective, injective and bijective mappings is respectively surjective, injective and bijective. **[2]**

END OF EXAMINATION PAPER