# MANICALAND STATE UNIVERSITY OF APPLIED SCIENCES

## FACULTY OF ENGINEERING

**DEPARTMENT OF MINING & MINERAL PROCESSING ENGINEERING**
**DEPARTMENT CHEMICAL & PROCESSING ENGINEERING**
**DEPARTMENT OF METALLURGICAL ENGINEERING**

**MODULE: ENGINEERING MATHEMATICS**

**CODE: ENGT 214**

**SESSIONAL EXAMINATIONS**

**DECEMBER 2023**

**DURATION: 3 HOURS**

**EXAMINER: MR W NKOMO**

***INSTRUCTIONS***

1. *Answer **All** in Section A*
2. *Answer **three** questions in Section B.*
3. *Start a new question on a fresh page*
4. *Total marks 100*

***Additional material(s):*** *Non-programmable electronic scientific calculator.*

## SECTION A [40 MARKS]

*Answer **ALL** questions in this section*

**QUESTION A1**

(a) Define the following terms as used in numerical analysis.

(i) True error [2]

(ii) Truncation error [2]

(b) How is absolute relative error used as a stopping criterion? [2]

(c) Why do we measure errors? [2]

**QUESTION A2**

a) Outline the bisection method algorithm. [5]

b) Solve $x - 2 - lnx = 0$ for the root nearest to 3, correct to 3 decimal places using the bisection method. [5]

**QUESTION A3**

(a) Evaluate $\int_0^6 \frac{dx}{2+x}$. [4]

(b) Approximate $\int_0^6 \frac{dx}{2+x}$ by using Simpson's $\frac{3}{8}^{th}$ rule with 7 ordinates, hence find the associated absolute relative true error, $|\epsilon_t|$. [8]

**QUESTION A4**

Solve the following system of equations using Gaussian elimination with partial pivoting: [10]

$$x - y + z = 1$$

$$-3x + 2y - 3z = -6$$

$$2x - 5y + 4z = 5$$

## SECTION B [60 MARKS]

*Answer any **THREE** questions in this section*

### QUESTION B1

(a) Outline two advantages and two drawbacks associated with Newton Raphson method in root estimation. [4]

(b) The equation that gives the depth $x$ (metres), to which an object is submerged under water is given by $f(x) = x^3 - 0.165x^2 + 3.993 \times 10^{-4}$. Show that a root of $f(x) = 0$ lies between 0 and 0.11.[3]

(c) Use Newton's method of finding roots of equations to find the depth *x* to which the object is submerged under water starting with $x_0$ =0.05. Conduct three iterations and find the absolute relative approximate error at the end of each iteration. [13]

### QUESTION B2

(a) Derive Newton Raphson method, $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$. [4]

(b) Use Newton's method to find the smallest positive root of the equation $tanx = x$ to within $10^{-4}$ for the initial approximation $x_0 = \frac{3\pi}{2}$. [6]

(c) Apply the Jacobi iterative method to approximate the solution of the following system of linear equations for an initial approximation $x_{1=}x_2 = x_3 = 0$. [10]

$$5x_1 - 2x_2 + 3x_3 = -1$$
$$-3x_1 + 9x_2 + x_3 = 2$$
$$2x_1 - x_2 - 7x_3 = 3$$

### QUESTION B3

(a) Derive the composite Trapezoidal rule: $\int_a^b fx\,dx \approx \frac{h}{2}\{f_0 + 2(f_1 + f_2 \ldots . f_{n-1}) + f_n\}$. [5]

(b) Show that the truncation error associated with (b) is $\frac{b-a}{12h^2} f^2(z)$. [3]

(c) Show that $\int_1^6 2 + \sin(\sqrt{x})\,dx = 8.183479$ hence investigate the absolute error when the composite Trapezoidal rule with 5 strips is used. [12]

**QUESTION B4**

(a) Apply the Gauss Seidel method to approximate the solution of the system of linear equations.

$$12x_1 + 3x_2 - 5x_3 = 1$$
$$x_1 + 5x_2 + 3x_3 = 28$$
$$3x_1 + 7x_2 + 13x_3 = 76,$$

with an initial guess $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. [10]

(b) A ball at 1200K is allowed to cool down in air at an ambient temperature of 300K. Assuming heat is lost only due to radiation, the differential equation for the temperature of the ball is given by $\frac{d\theta}{dt} = -2.2067 \times 10^{-12}(\theta^4 - 81 \times 10^8)$, $\theta(0) = 1200K$.

Find the temperature at t=480 seconds using Euler's method assuming a step size of h=240 seconds. [10]

************THE END************