MANICALAND STATE UNIVERSITY OF APPLIED SCIENCES

FACULTY OF ENGINEERING, APPLIED SCIENCES & TECHNOLOGY

DEPARTMENT OF APPLIED STATISTICS

MODULE: CALCULUS

CODE: ASTA 110

SESSIONAL EXAMINATIONS

DECEMBER 2023

DURATION: 3 HOURS

EXAMINER: MR W NKOMO

INSTRUCTIONS

1. *Answer **All** in Section A*
2. *Answer **three** questions in Section B.*
3. *Start a new question on a fresh page*
4. *Total marks 100*

Additional material(s): *Non-programmable electronic scientific calculator.*

List of Formulae

SECTION A [40 MARKS]

*Answer **ALL** questions in this section*

A1. Define the limit of a function $f(x)$, on an open interval c. [2]

A2. Given that $y = f(x) = \begin{cases} x^2, -1 \leq x \leq 0 \\ 1, \quad x = 0 \\ x^2, 0 \leq x \leq 1 \\ 0, \ 1 \leq x \leq 2 \end{cases}$,

a) Sketch the graph of $y = f(x)$. [4]

b) Using your graph, evaluate

i) $\lim_{x \to -1^+} f(x)$

ii) $\lim_{x \to 0^-} f(x)$

iii) $\lim_{x \to 0} f(x)$

iv) $\lim_{x \to 2^+} f(x)$ [2× 4]

c) Does $\lim_{x \to -1^-} f(x)$ exist? Justify. [3]

d) Verify that $\lim_{x \to 0^+} f(x) = \lim_{x \to o^+} f(x)$ [3]

A3. (a) Find a possible sequence U_n, whose first five terms are 1, 7, 17, 31, 49.[5]

(b) Find the first five (5) terms of the sequence, U_n, defined by $U_n = U_{n-1} + U_{n+2}$, $U \geq 3$ and $U_1 = U_2 = 1$. [5]

A4. (a) Evaluate $\frac{d}{dx}(x^x)$. [4]

(b) Show that $f(x) = sinx$ is differentiable everywhere and that

$f^{'}(x) = \cos(x)$. [6]

SECTION B [60 MARKS]

*Answer any **THREE** questions in this section*

B1. (a) Apply L 'Hospital's rule to prove that $\lim_{n \to \infty} \frac{\ln(n)}{n} = 0$. [5]

(b) Use Squeeze theorem to find the $\lim_{t \to 0} U(t)$ no matter how complicated U is

given that $2 - \frac{t^2}{5} \leq U_t \leq 2 + \frac{t^2}{3} \ \forall_t \neq 0.$ [6]

(c) Evaluate (i) $\frac{d}{dt}(\tanh(\sqrt{(}1 + t^2))$ [4]

(ii) $\int_0^{ln2} 4e^x \, sinhxdx$ [5]

B2. (a) Prove by induction that for all possible integers n,

$\frac{1}{2^1} + \frac{1}{2^2} \ldots\ldots\ldots + \frac{1}{2^n} = 1 - \frac{1}{2^{n.}}$ [7]

(b) Find $\frac{dy}{dx}$ if

(i) $y = \frac{1}{x}(x^2 + e^x)$ [3]

(ii) $y = \frac{t^2-1}{t^3+1}$ [3]

(iii) $y = cosu, \quad x = 1 + u^2$ [4]

(d) Find the set of values of x for which $\frac{1}{x-2} > \frac{2}{x+3}$ holds. [3]

B3. (a) Solve the differential equation $y(x+1)\frac{dy}{dx} = x(y^2+1)$ [8]

b) The biomass of a yeast culture in an experiment is initially 29 grammes. After 30 minutes, the mass is 37 grammes. Assuming the equation for unlimited population growth gives a good model for the growth of the yeast when the mass is below 100 grammes, how long will it take for the mass to double from its initial value? [12]

B4. (a) Find $\frac{dy}{dx}$ if $y = e^{2xy} + ysinx - 3xy^2 + 6$. [5]

(b) Find the equation of the normal to the curve $3x^2 - 7y^2 + 4xy - 8x = 0$ at the point (-1;1). [5]

(c) Find

(i) the area bounded by the x-axis and the curve $y = 4 - x^2$. [5]

(ii) the volume generated by revolving the region in part (i) about the x-axis. [5]

*********THE END**********