MANICALAND STATE UNIVERSITY OF

## APPLIED SCIENCES

## FACULTY OF ENGINEERING, APPLIED SCIENCES AND TECHNOLOGY

## DEPARTMENT OF MINING AND MINERAL PROCESSING ENGINEERING DEPARTMENT OF CHEMICAL AND PROCESSING ENGINEERING

## MODULE: ENGINEERING MATHEMATICS I

CODE: ENGT 102

## SESSIONAL EXAMINATIONS

DECEMBER 2023

DURATION: 3 HOURS<br>EXAMINER: NYAKUAMBA T

## INSTRUCTIONS

1. Answer All in Section A
2. Answer three questions in Section B.
3. Start a new question on a fresh page
4. Total marks 100

Additional material(s): Non-programmable electronic scientific calculator.

## SECTION A (40 marks)

## Answer ALL Questions

A1. a) You told that $\pi=\frac{22}{7}$, is this true or false. Give a reason for your answer.
[3]
b) Find the values of x for which the functions
(i) $\frac{4 x-3}{x(x+4)(2 x-9)}$
(ii) $\frac{1+\cos x}{1-\sin x}$

Are not defined.
A2 Let $f(x)=(x-2)(8-x)$ for $2 \leq x \leq 8$
(a) Find $f(6)$ and $f(-1)$.
(b) Determine the nature of the turning points and sketch the curve.

A3 (a). A farmer has an adjustable electric fence that is 100 m long. He uses it to enclose a rectangular grazing area on three, the fourth side being a fixed hedge. Find the maximum area he can enclose.
(b) differentiate $\frac{1}{x^{2}}$ from the first principals.

A4. Differentiate the following functions with respect to $x$
(i) $f(x)=x^{2}-\frac{1}{\sqrt{x}}+\ln x$
(ii) $x^{2}-x y+y^{2}=0$

$$
\begin{equation*}
\text { (iii) } \quad x=\cos 2 t ; \quad y=\sin 2 t \tag{3}
\end{equation*}
$$

A5) Find the set of valves of $x$ for which the following set of inequalities hold

$$
\begin{align*}
& \text { (a) } x^{2} \geq 5 x-6  \tag{4}\\
& \text { (b) } \frac{1}{x-2}>\frac{2}{x+3}
\end{align*}
$$

## SECTION B. (60 Marks)

Candidate may attempt three questions being careful to number them B6 to B9

B6.(i) Evaluate the following limits
a) $\lim _{n \rightarrow \infty} \frac{3 n^{3}+n-2}{2 n^{3}-n^{2}+6}$
b) $\lim _{n \rightarrow \infty}\left(\frac{n+2}{3 n-5}\right)^{3}$
c) $\lim _{n \rightarrow 0} \frac{\sin x}{x}$
d) $\lim _{n \rightarrow \infty}(\sqrt{n+10}-\sqrt{n})$
[4]
(ii)Solve the following equations
(a) $|3+2 x|=2|x+1|$
(b) $\frac{2}{7 x}-\frac{4}{3 x}>1$
$B 7$ (a) Differentiate the following functions with respect to $x$
(i) $y=3 x^{2}+2 x+7+e^{3 x^{2}-3 x+6}$
(ii) $y=\frac{e^{x}}{e^{x}-e^{-x}}$
(iii) $y=\frac{1}{x^{2}}$
b)Integrate the following functions with respect to $x$.
i) $\frac{\operatorname{Cos} x-\operatorname{Sin} x}{\sin x+\cos x}$
ii) $3 e^{-3 x}-\frac{1}{2} e^{2 x}$
iii) $x^{2} e^{x}$
iv) $\ln x$
$B 8 a)$ Find the area of the bounded plane $R$ lying between the curves
$y=x^{2}-4 x$ and $y=4-x^{2}$.
b) Given that $x=3(2 \theta-3 \operatorname{Sin} 2 \theta)$ and $y=3(1-\operatorname{Cos} 2 \theta)$

$$
\text { Find } \frac{d y}{d x}
$$

[5]
c) Find the equation of the tangent to the curve
$3 x^{2}-7 y^{2}+4 x y-8 x=0$ at the point $(-1,1)$.
d)(i) Define cosh $x$ and $\sinh x$ in terms of exponentials.
(ii) Using the definition in (i) above show that
$\frac{d}{d x} \cosh (x)=\sinh (x)$
B 9. (a) Prove by induction that
$\sum_{r=1}^{n} r^{2}=\frac{n}{6}(n+1)(2 n+1)$ for all $n \in \mathcal{R}$
(b) (i) Integrate $x^{2} e^{x}$ with respect to $x \quad$ [3]
(ii) Express $\frac{2 x-3}{x^{2}-5 x+6}$ in partial fractions hence or otherwise

$$
\int_{0}^{1} \frac{2 x-3}{x^{2}-5 x+6} \mathrm{dx} \quad[4,6]
$$

## END OF QUESTION PAPER

