

MANICALAND STATE UNIVERSITY OF APPLIED SCIENCES

FACULTY OF ENGINEERING, APPLIED SCIENCES AND TECHNOLOGY

DEPARTMENT OF MINING AND MINERAL PROCESSING ENGINEERING DEPARTMENT OF CHEMICAL AND PROCESSING ENGINEERING

MODULE: ENGINEERING MATHEMATICS I

CODE: ENGT 102

SESSIONAL EXAMINATIONS

DECEMBER 2023

DURATION: 3 HOURS

EXAMINER: NYAKUAMBA T

INSTRUCTIONS

- 1. Answer All in Section A
- 2. Answer three questions in Section B.
- 3. Start a new question on a fresh page
- 4. Total marks 100

Additional material(s): Non-programmable electronic scientific calculator.

SECTION A (40 marks)

Answer ALL Questions

A1. a) You told that $\pi = \frac{22}{7}$, is this true or false. Give a reason for your answer.

[3]

b) Find the values of x for which the functions

(i)
$$\frac{4x-3}{x(x+4)(2x-9)}$$

(ii)
$$\frac{1+\cos x}{1-\sin x}$$

Are not defined. [3,2]

A2 Let f(x) = (x - 2)(8 - x) for $2 \le x \le 8$

(a) Find
$$f(6)$$
 and $f(-1)$. [3]

(b) Determine the nature of the turning points and sketch the curve. [5]

A3 (a). A farmer has an adjustable electric fence that is 100m long. He uses it to enclose a rectangular grazing area on three, the fourth side being a fixed hedge. Find the maximum area he can enclose. [5]

(b) differentiate
$$\frac{1}{x^2}$$
 from the first principals. [3]

A4. Differentiate the following functions with respect to x

(i)
$$f(x) = x^2 - \frac{1}{\sqrt{x}} + \ln x$$
 [2]

(ii)
$$x^2 - xy + y^2 = 0$$
 [3]

(iii)
$$x = cos2t$$
; $y = sin2t$ [3]

A5) Find the set of valves of x for which the following set of inequalities hold

$$(a)x^2 \ge 5x - 6 \tag{4}$$

(b)
$$\frac{1}{x-2} > \frac{2}{x+3}$$
 [4]

SECTION B. (60 Marks)

Candidate may attempt three questions being careful to number them B6 to B9

B6.(i) Evaluate the following limits

a)
$$\lim_{n\to\infty} \frac{3n^3+n-2}{2n^3-n^2+6}$$
 [3]

b)
$$\lim_{n \to \infty} \left(\frac{n+2}{3n-5}\right)^3$$
 [3]

c)
$$\lim_{n \to 0} \frac{\sin x}{x}$$
 [2]

d)
$$\lim_{n \to \infty} (\sqrt{n+10} - \sqrt{n})$$
 [4]

(ii)Solve the following equations

(a)
$$|3 + 2x| = 2|x + 1|$$
 [4]

(b)
$$\frac{2}{7x} - \frac{4}{3x} > 1$$
 [4]

B7(a) Differentiate the following functions with respect to x

(i)
$$y = 3x^2 + 2x + 7 + e^{3x^2 - 3x + 6}$$
 [4]

(ii)
$$y = \frac{e^x}{e^x - e^{-x}}$$
 [4]

(iii)
$$y = \frac{1}{r^2}$$
 [2]

b)Integrate the following functions with respect to x.

i)
$$\frac{\cos x - \sin x}{\sin x + \cos x}$$

ii)
$$3e^{-3x} - \frac{1}{2}e^{2x}$$

- iii) x^2e^x
- iv) lnx

B8 a) Find the area of the bounded plane R lying between the curves $y = x^2 - 4x$ and $y = 4 - x^2$. [5]

b) Given that $x = 3(2\theta - 3Sin 2\theta)$ and $y = 3(1 - Cos 2\theta)$

Find
$$\frac{dy}{dx}$$
 [5]

c)Find the equation of the tangent to the curve

$$3x^2 - 7y^2 + 4xy - 8x = 0$$
 at the point(-1,1). [5]

d)(i) Define cosh x and sinhx in terms of exponentials.

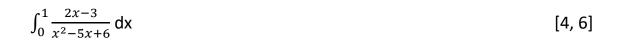
(ii)Using the definition in (i) above show that

$$\frac{d}{dx}\cosh(x) = \sinh(x) \tag{5}$$

B 9. (a) Prove by induction that

$$\sum_{r=1}^{n} r^2 = \frac{n}{6} (n+1)(2n+1) \text{ for all } n \in \mathcal{R}$$
 [7]

- (b) (i) Integrate x^2e^x with respect to x [3]
- (ii) Express $\frac{2x-3}{x^2-5x+6}$ in partial fractions hence or otherwise



END OF QUESTION PAPER