MANICALAND STATE UNIVERSITY OF APPLIED SCIENCES

FACULTY OF ENGINEERING, APPLIED SCIENCES AND TECHNOLOGY

DEPARTMENT OF APPLIED STATISTICS, DEPARTMENT OF COMPUTER SCIENCE

MODULE: CALCULUS 1

CODE: ASTA 102/ASTA110

MODULE: CALCULUS

CODE; ASTA 110

SESSIONAL EXAMINATIONS

DECEMBER 2023

DURATION: 3 HOURS

EXAMINER: NYAKUAMBA T

INSTRUCTIONS

1. *Answer **All** in Section A*
2. *Answer **three** questions in Section B.*
3. *Start a new question on a fresh page*
4. *Total marks 100*

Additional material(s): *Non-programmable electronic scientific calculator.*

SECTION A (40 marks)

Answer ALL Questions

A1. a) What do we mean when we say a function is continuous at x_0 [3]

b) Prove that $f(x) = x^2$ is continuous at $x = 2$ [5]

A2 Let $f(x) = \begin{cases} \frac{|x-3|}{x-3} & ; x \neq 3 \\ 0 \; ; x = 3 \end{cases}$

(a) Graph the function $f(x)$ [3]

(b) Find the $\lim_{x \to 3^+} f(x)$ [2]

(c) Find the $\lim_{x \to 3^-} f(x)$ [2]

(d) Find the $\lim_{x \to 3} f(x)$ [1]

A3 Evaluate each of the following limits

(a). $lim_{(x,y)\to(0,0)} \frac{e^y sinx}{x}$ [4]

(b). $lim_{(x,y)\to(0,0)} \frac{3x^2-y^2+5}{x^2+y^2+2}$ [4]

A4. Differentiate the following functions with respect to x

(i) $f(x) = x^2 - \frac{1}{\sqrt{x}} + lnx$ [2]

(ii) $x^2 - xy + y^2 = 0$ [3]

(iii) $x = cos2t; \quad y = sin2t$ [3]

A5) Find the set of valves of x for which the following set of inequalities hold

(a) $2x^2 - 3x - 5 \geq 0$ [4]

(b) $\frac{1}{x-2} > \frac{2}{x+3}$ [4]

SECTION B. (60 Marks)

Candidate may attempt three questions being careful to number them B6 to B9

B6.(i) Evaluate the following limits

a) $\lim_{n\to\infty}(\frac{1-\cos x}{x^2})$ [2]

b) $\lim_{n\to 2}(\frac{3n^4-8n^3+16}{x^3-3x^{2+4}})$ [2]

c) $\lim_{n\to 0}\frac{sinx}{x}$ [2]

d) $\lim_{x\to 5} 3$ [2]

e) $\lim_{n\to\infty}(\sqrt{n+10}-\sqrt{n})$ [4]

(ii)Solve the following equations

(a) $|3 + 2x| = 2|x + 1|$ [4]

(b) $\frac{2}{7x} - \frac{4}{3x} > 1$ [4]

B7(a) Differentiate the following functions with respect to x

(i) $y = 3x^2 + 2x + 7 + e^{3x^2-3x+6}$ [4]

(ii) $y = \frac{e^x}{e^x-e^{-x}}$ [4]

(iii) $y = \frac{1}{x^2}$ [2]

b) Integrate the following functions with respect to x.

i) $\frac{Cos\ x - Sin\ x}{Sin\ x + Cos\ x}$

ii) $3e^{-3x} - \frac{1}{2}e^{2x}$

iii) $(3x+5)^5$

iv) $Cos(6-7x)$ [10]

B8 a) Evaluate $\iiint_Q f(x,y,z)dV$ Were

$$Q = \{(x,y,z): -1 \leq x \leq 3, 1 \leq y \leq 4, 0 \leq z \leq 2\}$$ [6]

b) Given that $x = 3(2\theta - 3Sin\ 2\theta)$ and $y = 3(1 - Cos\ 2\theta)$

Find $\frac{dy}{dx}$ [4]

c)Find the equation of the tangent to the curve
$3x^2 - 7y^2 + 4xy - 8x = 0$ at the point$(-1,1)$. [5]

d)(i) Define $\cosh x\ and\ \sinh x$ in terms of exponentials.

(ii)Using the definition in (i) above show that

$\frac{d}{dx}\cosh(x) = \sinh(x)$ [5]

B 9. (a) Prove by induction that

$\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$ for all $n \in \mathcal{R}$ [7]

(b) (i) Integrate x^2e^x with respect to x [3]

(c) Find (i) the area bounded by the X-axis and the curve $y = 4 - x^2$

(ii) the volume generated by revolving the region in part (i) about

the X-axis [3,3]

(d)Find the length of an arc of the parabola $y = x^2$ from $x = 0\ to\ x = 1$ [4]

END OF QUESTION PAPER