MANICALAND STATE UNIVERSITY OF

## APPLIED SCIENCES

## FACULTY OF APPLIED SCIENCES \& TECHNOLOGY DEPARTMENT OF COMPUTER SCIENCE \& INFORMATION SYSTEMS

## MODULE: COMPUTATIONAL MATHEMATICS FOR <br> INFORMATION SYSTEMS

CODE: INSY121
SESSIONAL EXAMINATIONS
APRIL 2023

DURATION: 3 HOURS
EXAMINER: DR. W. GOVERE

## INSTRUCTIONS

1. Answer All in Section A
2. Answer three questions in Section $B$.
3. Start a new question on a fresh page
4. Total marks 100

Additional material(s): Non-programmable electronic scientific calculator.

## SECTION A (ANSWER ALL QUESTIONS) [40 MARKS]

A1
Solve the following system of linear equations by Gauss-Jordan elimination method:

$$
\begin{gathered}
2 x+y-z=-1 \\
-2 x+y+2 z=1 \\
x+y+z=2
\end{gathered}
$$

## A2.

Suppose the array $\left[\begin{array}{lll}4 & 3 & 3 \\ 2 & 1 & 0 \\ 4 & 4 & 2\end{array}\right]$ represents the orders placed by three individuals at Nandos Samora Macheal Avenue/ $6^{\text {th }}$ Street. The first-person orders 4 burgers, 3 pet drinks, and 3 fries; the second orders 2 burgers and 1 pet drink, and the third orders 4 burgers, 4 pet drinks, and 2 fries, burgers cost $\$ 2$ each, pet drinks $\$ 1$ each and fries $\$ 1.50$ each.
(a) Show that the amounts owed by these persons may be represented as a function $y=f(x)$, where $f(x)$ is equal to the array given above times a certain vector.
(b) Compute the amounts owed in this case by performing the appropriate multiplication.
(c) Change the matrix for the case in which the second person orders an additional pet drink and 2 fries, and recompute the costs.

National Foods Stockfeeds company markets two types of feed for cattle. The first mix, Fertilex, requires at least twice the amount of wheat as barley. The second mix, Multiplex, requires at least twice the amount of barley as wheat. Wheat costs $\$ 2.50$ per kg and 1000 kg is available per month. Barley costs $\$ 2.25$ per kg and 1200 kg is available per month. Fertilex sells for $\$ 2.80$ per kg up to 99 kg and every additional kilogram sells for $\$ 2.65$. Multiplex sells for $\$ 2.70$ per kg up to 99 kg and each additional kilogram sells for $\$ 2.55$. CSC farms will buy any and all amounts of both mixes of National Foods Stockfeeds company. Formulate the mathematical model for this problem.

## A4

A manufacturer of Puma clothing makes Puma pants and Puma jackets. The profit on a pair of Puma pants is $\$ 2.00$ and on a Puma jacket is $\$ 1.50$. Both pants and jackets require the work of sewing operators and cutters. There are 60 minutes of sewing operator time and 48 minutes of cutter time available. It takes 8 minutes to sew one pair of Puma pants and 4 minutes to sew one Puma jacket. Cutters take 4 minutes on pants and 8 minutes on a jacket. Find the maximum profit and the amount of pants and jackets to maximize the profit.
a) Let $x=$ Puma pants and $y=$ Puma jackets. Since there cannot be negative pants or jackets, write two inequalities to represent that situation.
b) Express the cutters' time to make pants and jackets as an inequality.
c) Express the sewing operators' time to make pants and jackets as an inequality.
d) Write an equation for the anticipated profit.
e) Graph the constraints.
f) Use the corner points to find the maximum profit.
g) What is the maximum profit?
h) How many Puma pants and Puma jackets have to be made to maximize profit?
$[3,1,1,2,6,3,2,2]$

## SECTION B (ANSWER ANY THREE QUESTIONS) [60 MARKS]

## B5

a) If $\mathrm{A}=\left(\begin{array}{ccc}1 & 4 & -2 \\ 2 & 5 & 1 \\ -1 & 3 & -3\end{array}\right)$
i) Evaluate $|A|$ and hence find $A^{-1}$.
ii) Evaluate $A^{2}-2 A$.
iii) Use the method of inverses to solve $A X=(2,5,-1)^{T}$.
iv) Prove that $\operatorname{det}\left(A^{-1}\right)=\frac{1}{\operatorname{det}(A)}$.
b) Find the value(s) of $a$ and $b$ for which the systems of equations

$$
\begin{aligned}
& y+3 z=2 \\
& x+y-2 z=5 \\
& x+y+a z=3 b
\end{aligned}
$$

have
i) a unique solution
ii) no solution
iii) many solutions.

Each coffee table produced by T.V Sales Designers nets the firm a profit of $\$ 9$. Each bookcase yields a $\$ 12$ profit. T.V Sales is a small firm, and its resources are limited. During any given production period (of one week), 10 litres of varnish and 12 lengths of high quality redwood are available. Each coffee table requires approximately 1 litres of varnish and 1 length of redwood. Each bookcase takes 1 litres of varnish and 2 lengths of wood.
(a). Formulate TV Sales production mix decision as a linear programming problem.
(b). Solve this problem using the Simplex method. Determine how many tables and bookcases should be produced each week?
(c). What would be the maximum profit?
$[5,10,5]$

## B7

(a) Use the graphical solution technique to find the optimal solution to the model below:

Maximize $Z=10 x_{1}+20 x_{2}$
subject to

$$
\begin{gathered}
2 x_{1}+x_{2} \leq 40 \\
5 x_{1}-2 x_{2} \leq 20 \\
x_{1} \geq 25 \\
x_{1}, x_{2} \geq 0
\end{gathered}
$$

(b)A computer centre has five expert programmers. The centre wants three application programs to be developed. The head of the computer centre, after carefully studying the programs to be developed, estimates the time (in minutes) required by the experts for developing the application programs as given in the following table:

| Program | Programmer |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\boldsymbol{A}$ | $\boldsymbol{B}$ | $\mathbf{C}$ | $\mathbf{D}$ | $\boldsymbol{E}$ |
| $\boldsymbol{I}$ | 120 | 100 | 80 | 90 | 130 |
| $\boldsymbol{I I}$ | 80 | 90 | 110 | 70 | 110 |
| III | 110 | 140 | 120 | 130 | 160 |

Assign the programmers to the programs in such a way that the minimum total time is taken for developing the programs.
$[5,15]$

## B8

A product is produced by four factories $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D . Their unit production costs are $\$ 2, \$ 3, \$ 1$, and $\$ 5$ respectively. The production capacities in each factory are 50 units, 70 units, 30 units and 50 units respectively. These factories supply the units to four stores $P, Q, R$, and $S$, the demands of which are $25,35,105$, and 20 units, respectively. Unit transportation cost in dollars from each factory to each store is given in the following table:

| Factories | Stores |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | P | $\mathbf{Q}$ | $\mathbf{R}$ | $\mathbf{S}$ |  |
| A | 2 | 4 | 6 | 11 |  |
| B | 10 | 8 | 7 | 5 |  |
| C | 13 | 3 | 9 | 12 |  |
| D | 4 | 6 | 8 | 3 |  |

Determine the extent of deliveries from each of the factories to each of the stores so that the total transportation and the production cost is minimum.
[20]

## END OF QUESTION PAPER

