MANICALAND STATE UNIVERSITY OF APPLIED SCIENCES

FACULTY OF ENGINEERING, APPLIED SCIENCES & TECHNOLOGY

DEPARTMENT OF MINING & MINERAL PROCESSING ENGINEERING
DEPARTMENT OF CHEMICAL & PROCESSING ENGINEERING
DEPARTMENT OF METALLURGICAL ENGINEERING

MODULE: ENGINEERING MATHEMATICS V

CODE: ENGT 314

SESSIONAL EXAMINATIONS

APRIL 2023

DURATION: 3 HOURS

EXAMINER: DR W. GOVERE

INSTRUCTIONS

1. *Answer **All** in Section A*
2. *Answer **three** questions in Section B.*
3. *Start a new question on a fresh page*
4. *Total marks 100*

Additional material(s): *Non-programmable electronic scientific calculator, List of formulae.*

SECTION A: [40 MARKS]

*Answer **all** questions in this section*

A1.

(a) Prove that if B and C are both inverses of the matrix A, then $B = C$.

(b) Assuming that the sizes of the matrices are such that the indicated operations can be performed prove that $A(B + C) = AB + AC$.

(c) Prove that

$$|A^{-1}| = \frac{1}{\det(A)}.$$

(d) Prove that if $ad - bc \neq 0$, then the reduced row echelon form of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

(e) Use the result in part (d) to prove that if $ad - bc \neq 0$, then the linear system

$$ax + by = k$$
$$cx + dy = l,$$

has exactly one solution.

(f) Define the terms ordinary and regular singular points in connection with a generic second order linear differential equation.

(g) State Frobenius' Theorem for solution of differential equations about a regular singular point.

[3, 3, 3, 3, 3, 3, 3]

A2

(a) Find the values of x and y in the equation $(x + iy)(2 + i) = 3 - i$, given further that $x, y \in \mathbb{R}$.

(b) The cubic equation $2z^3 - 5z^2 + cz - 5 = 0 \quad c \in \mathbb{R}$, has a solution $z = 1 - 2i$ and hence find in any order:

(i) the other two solutions of the equation.

(ii) the value of c.

[3, 4]

A3

a) Use the transformation $xy = t$ to solve the equation

$$(1 + x^2y^2)y + (xy - 1)^2 xy' = 0$$

b) Solve the Bernoulli equation

$$xy' + y = x^2y^2$$

c) By finding an integrating factor, solve the initial value problem

$$(3x^2 + 8y)dy + 2xydx = 0, \qquad y(0) = 1$$

[4, 4, 4]

SECTION B: [60 MARKS]

Answer any <u>three</u> questions in this section

B4

(a) Let $A = \begin{bmatrix} 4 & 4k & k \\ 0 & k & 4k \\ 0 & 0 & 4 \end{bmatrix}$ be a given matrix. If the $|A^2| = 16$, find the value of k

(b) Using row reduction, show that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (b - a)(c - a)(c - b).$$

(c) Solve the following systems of nonlinear equations for x, y and z:

$$x^2 + y^2 + z^2 = 6$$
$$x^2 - y^2 + 2z^2 = 2$$
$$2x^2 + y^2 - z^2 = 3$$

(d) Find the value of a & b for which the following systems of equations

$$x - y + 2z = 4$$
$$3x - 2y + 9z = 14$$
$$2x - 4y + az = b$$

have

i. no solution.
ii. unique solution.
iii. infinitely many solutions.

[4, 4, 5, 3, 2, 2]

B5

(a) Show that the equation,

$$\frac{dy}{dx} + \frac{ax + hy + g}{hx + by + f} = 0$$

is an exact first-order equation and hence solve it.

(a) Prove that for a $y'' + p(x)y' + q(x)y = 0$, any linear combination of two solutions on an open interval I is again a solution of $y'' + p(x)y' + q(x)y = 0$ on I. In particular prove that, for such an equation, sums and constant multiples of solutions are again solutions.

(b) Solve the differential equation

$$2x\sin y - y\sin x + \left(x^2\cos y + \cos x\right)y' = 0$$

subject to boundary conditions $y = \frac{\pi}{6}$ if $x = \frac{\pi}{2}$

(c) Given that $y_1 = e^{2x}$ is a solution of $y'' - 4y' + 4y = 0$ on the interval $(-\infty, \infty)$, use reduction of order to find a second solution y_2.

(d) Using the method of undetermined coefficients find the general solution of $9\frac{d^2x}{dt^2} + 18\frac{dx}{dt} - 16x = 18\cos 7t$.

[5, 3, 4, 4, 4]

B6

a) Derive a general expression for the Laplace transform of $\frac{d^n y}{dx^n}$.

Hence solve the following Laplace transform

$$y''' + y' = e^x, \qquad y(0) = y'(0) = y''(0) = 0.$$

b)

i) Show that any Cauchy-Euler second order differential equation can be reduced to any equation with constant coefficients by means of the substitution

$$x = e^t \ .$$

ii) Hence or otherwise find the general solution of the Euler equation

$$x^2y'' - 2xy' - 4y = 0$$

[9, 6, 5]

B7

a) Determine the convergence set for

$$\sum_{n=0}^{\infty} \frac{2^{-n}}{n+1}(x-1)^n$$

b) Write

$$\sum_{n=2}^{\infty} n(n-1)c_n x^{n-2} + \sum_{n=0}^{\infty} c_n x^{n+1}$$

as one power series.

c) Show that $x = 0$ is a regular singular point of the equation

$$4xy'' + 2y' + y = 0$$

and obtain its solution near $x = 0$.

d) Solve the differential equation

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

Using the variation of parameters method

[4, 3, 9, 4]

END OF QUESTION PAPER