MANICALAND STATE UNIVERSITY OF APPLIED SCIENCES

FACULTY OF APPLIED SCIENCES & TECHNOLOGY

DEPARTMENT OF APPLIED STATISTICS

MODULE: ESTIMATION TECHNIQUES

CODE: ASTA 214

SESSIONAL EXAMINATIONS

JUNE 2023

DURATION: 3 HOURS

EXAMINER: MR M. TSODODO

INSTRUCTIONS

1. *Answer **All** in Section A*
2. *Answer **two** questions in Section B.*
3. *Start a new question on a fresh page*
4. *Total marks 100*

Additional material(s): *Non-programmable electronic scientific calculator.*
Statistical tables

SECTION A

Question 1

a) What properties do we consider in coming up with the best point estimators of population parameters. [3]

b) Name any four methods that can be used as point estimators. [4]

c) Let $x_1, \ldots \ldots . . x_5$ be the realised sample values of random samples $X_1, \ldots \ldots . . X_5$ where the random variables are drawn from a population with mean μ. Find,

i) An estimator of μ.

ii) An estimate of μ. [5]

Question 2

a) In Signal Detection Theory explain any six of the following terms citing examples if necessary from a given scenario where cars are passing through traffic lights (robots).

i) Signal

ii) Noise

iii) Hits

iv) Misses

v) False alarms

vi) Correct Rejections

vii) Conservative method

viii) Liberal method [3]

b) Illustrate in a table the use of the first six terms in a) above [4]

c) Design a test that maximizes the probability of detection

$$P_D = P[X \epsilon x_1; \theta = \theta_0]$$ [4]

Question 3

Let $X_1, X_2, \ldots \ldots \ldots . . X_n$ be a random variable from the Poisson distribution

$$p(x, \lambda) = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, \ldots$$

a) Show that $E(X) = \lambda$. [2]

b) Find the estimator λ using maximum likelihood method. [4]

c) Using the Poisson distribution above prove that $\overline{x}$

i) Is an unbiased estimator of λ. [2]

ii) Is a consistent estimator of λ. [4]

d) Use the observed values $\{x_1, \ldots \ldots, x_{12}\} = \{4, 2, 5, 8, 4, 4, 6, 3, 2, 4, 10, 2\}$ to find an estimator of λ. [2]

Question 4

a) Let two variables X and Y be related through the model $Y_i = \beta_0 + \beta_1 X_1 + \epsilon$, where $\epsilon \sim N(0, \sigma^2)$

   i) Find the least squares estimates of β_0 and β_1 [4,4]

   ii) Show that $\hat{\beta}_0$ and $\hat{\beta}_1$ are unbiased estimators of β_0 and β_1 [5,5]

SECTION B

Question 5

a) The following amounts in millions of dollars represent a company's expenditure on salaries over a period of 7 years. Data for 2 years, namely the first and the seventh years, are missing.

Year (x)	1	2	3	4	5	6	7
Expenditure(y)	*	20	25	26	29	30	*

   i) Plot the data and comment on the main features of the data. [5]

   ii) Estimate the trend function by fitting the least squares line $U_{(x)} = a + bx$. Hence, find estimates of expenditure on salaries for the two years for which figures are missing. [10]

b) Let $X_1, \dots \dots \dots \dots X_n$ be a random sample from the geometric distribution $P(x, \theta) = \left(1 - {}^{1}\!/\!{}_{\theta}\right)\left({}^{1}\!/\!{}_{\theta}\right)^x, \; x = 0, 1, 2, \dots \dots \dots \dots$

   Find the least squares estimator for θ. [10]

c) Let $X_1, \dots \dots \dots . X_n$ be a random sample from a normal distribution $N(\theta, 1)$. Prove that $\hat{\theta} = \bar{X}$ is a consistent estimator of θ. [5]

Question 6

a) State the Cramer-Rao Lower Bound Theorem and its conditions to hold [10]

b) Let $X_1, \dots \dots \dots \dots . . X_n$ be a random sample from an exponential distribution with parameter θ, that is $f_{x_i}(x_i; \theta) = \theta e^{-\theta x_i}$. Suggest a possible sufficient statistics. [5]

c) Let $X_1, \dots \dots \dots . . X_n$ be a random sample obtained from an exponential distribution with parameter θ.

   i) Find the lower bound of the variance of unbiased estimators of $\tau(\theta) = {}^{1}\!/\!{}_{\theta} = E(X)$. [10]

   ii) Is $\overline{X}$ an UMVUE of $E(X)$? [5]

d)

Question 7

a) Let $X_1, X_2, \ldots, X_n$ be a random sample from a Bernoulli probability mass function (p.m.f) given by f(x|θ)= $\theta^x(1-\theta)^{1-x}$

$x = 0,1$

Assume a uniform distribution for θ is $p_\theta(\theta)$ =1, $\theta \in (0, 1)$.

Find the Posterior Bayes Estimator of θ [10]

b) Let $X_1, \ldots \ldots \ldots . . X_n$ be a random sample from a Poisson distribution with unknown parameter θ, that is. $P(X = x) = f_X(x; \lambda) = \frac{e^{-\theta}\theta^x}{x!} . I_{\{0,1,\ldots\}}(x)$ where $\lambda \geq 0$.

i) Assuming a uniform prior density θ, find the posterior distribution of θ. [10]

ii) Assuming a gamma prior density for θ,
Find the Bayes' estimator of θ [5]
Find the posterior Bayes' estimator of $\tau(\theta) = P[X_i = 0]$ [5]

END OF EXAMINATION