



**MANICALAND STATE UNIVERSITY
OF
APPLIED SCIENCES**

**FACULTY OF ENGINEERING, APPLIED SCIENCES AND
TECHNOLOGY**

DEPARTMENT OF APPLIED STATISTICS

MODULE: CALCULUS I

CODE: ASTA 102

**SESSIONAL EXAMINATIONS
JUNE 2023**

DURATION: 3 HOURS

EXAMINER: MR TSODODO

INSTRUCTIONS

1. Answer **All** in Section A
2. Answer **three** questions in Section B.
3. Start a new question on a fresh page
4. Total marks 100

Additional material(s): Non-programmable electronic scientific calculator.

SECTION A (40 marks)

Answer ALL Questions

A1. a) Differentiate $\frac{1}{x}$ from the first principals [3]

b) For what values of x is each of the following functions continuous

(i) $f(x) = \frac{2x-3}{(3x+7)(x-4)}$ [2]

(ii) $f(x) = \frac{1+\cos x}{3+\sin x}$ [1]

(iii) $f(x) = \frac{x-|x|}{x}$ [2]

A2 Let $f(x) = \begin{cases} \frac{|x-3|}{x-3} & ; x \neq 3 \\ 0 & ; x = 3 \end{cases}$

(a) Graph the function $f(x)$ [3]

(b) Find the $\lim_{x \rightarrow 3^+} f(x)$ [2]

(c) Find the $\lim_{x \rightarrow 3^-} f(x)$ [2]

(d) Find the $\lim_{x \rightarrow 3} f(x)$ [1]

A3. (a). State the second fundamental theorem of calculus [1]

(b). Find $fg(x)$ and $gf(x)$ in the following:

(i) $f(x) = x + 2$ $g(x) = x^2 - 4$ [2]

(ii) $f(x) = \frac{x-1}{x+1}$ $g(x) = \frac{1}{x}$ [3]

(c). You are told that $\pi=22/7$ is this true or false. Give reason for your answer

[2]

A4. Differentiate the following functions with respect to x

a) $f(x) = x^2 - \frac{1}{\sqrt{x}} + \ln x$ [2]

b) $x^2 - xy + y^2 = 0$ [3]

c) $x = \cos 2t; y = \sin 2t$ [3]

A5) Find the set of values of x for which the following set of inequalities hold

(a) $2x^2 - 3x - 5 \geq 0$ [4]

(b) $\frac{1}{x-2} > \frac{2}{x+3}$ [4]

SECTION B. (60 Marks)

Attempt three questions being careful to number them B6 to B9

B6.(a) Evaluate the following limits

i. $\lim_{n \rightarrow \infty} \frac{n^3 + 4n - 5}{n^2 - 1}$ [2]

ii. $\lim_{n \rightarrow \infty} \left(\frac{2n-7}{5n+4}\right)^4$ [2]

iii. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$ [2]

iv. $\lim_{x \rightarrow 5} 3$ [2]

v. $\lim_{n \rightarrow \infty} (\sqrt{n+10} - \sqrt{n})$ [4]

(b) Solve the following equations

i. $|3 + 2x| = 2|x + 1|$ [4]

ii. $\frac{2}{7x} - \frac{4}{3x} > 1$ [4]

B7(a) Differentiate the following functions with respect to x

(i) $y = 3x^2 + 2x + 7 + e^{3x^2 - 3x + 6}$ [4]

(ii) $x = t - \frac{1}{t}$ and $y = \frac{1}{t^2}$ [4]

(iii) $y = \frac{1}{x^2}$ [2]

b) Integrate the following functions with respect to x .

i) $\frac{\cos x - \sin x}{\sin x + \cos x}$

ii) $3e^{-3x} - \frac{1}{2}e^{2x}$

iii) $(3x + 5)^5$

iv) $\text{Cos}(6 - 7x)$ [10]

B8

a) Find the area of the bounded plane region R lying between the curves
 $y = x^2 - 2x$ and $y = 4 - x^2$ [5]

b) Given that $x = 3(2\theta - 3\text{Sin } 2\theta)$ and $y = 3(1 - \text{Cos } 2\theta)$
Find $\frac{dy}{dx}$ [5]

c) Find the equation of the tangent to the curve
 $3x^2 - 7y^2 + 4xy - 8x = 0$ at the point $(-1, 1)$. [5]

d)
i. Define $\cosh x$ and $\sinh x$ in terms of exponentials.
ii. Using the definition in (i) above show that $\frac{d}{dx} \cosh(x) = \sinh(x)$ [5]

B 9.

a) Deduce the formula for the sum $\frac{1}{1.2} + \frac{1}{2.3} + \dots + \frac{1}{n(n+1)}$ and prove it by
induction [7]

b)
i. Integrate $x^2 e^x$ with respect to x [3]

i. Express

$$\frac{2x - 3}{x^2 - 5x + 6}$$

in partial fractions hence or otherwise

$$\int_0^1 \frac{2x - 3}{x^2 - 5x + 6} dx$$

[4, 6]

END OF QUESTION PAPER