MANICALAND STATE UNIVERSITY OF APPLIED SCIENCES

FACULTY OF ENGINEERING, APPLIED SCIENCES & TECHNOLOGY

DEPARTMENT OF COMPUTER SCIENCE

MODULE: DISCRETE MATHEMATICS

CODE: BCOS 121

SESSIONAL EXAMINATIONS

APRIL 2023

DURATION: 3 HOURS

EXAMINER: D. MHINI

INSTRUCTIONS

1. *Answer **All** in Section A*
2. *Answer **three** questions in Section B.*
3. *Start a new question on a fresh page*
4. *Total marks 100*

***Additional material(s):** Non-programmable electronic scientific calculator.*

SECTION A: Answer ALL in this section [40]

A1. (a) Prove that $2^0 = 1$ **[3]**

(b) Define a logical equivalence and show that

$p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$ **[5]**

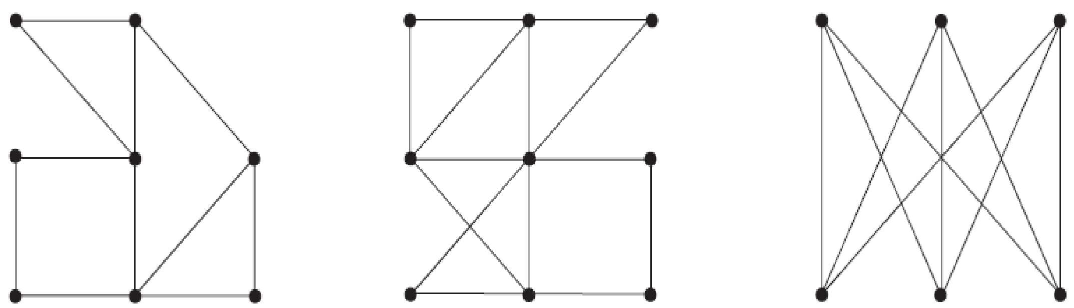
(c) Show that $[(p \rightarrow q) \wedge (p \rightarrow r)] \rightarrow (p \wedge r)$ is a Tautology **[5]**

A2.(a) A license plate is to consist of three letters followed by two digits. How many different license plates are possible if the first letter must be a vowel $(a, e, i, o, u\ )$ and repetition of letters is not permitted, but repetition of digits is permitted? **[5]**

(b). Show that $\sqrt{3}$ is irrational. **[5]**

(c). Present a direct proof : $2^n \leq n! \leq n^n$ **[8]**

A3 Consider each graph below. Which of them are traversable that is have Euler path? Which are Eulerian that is, have an Euler circuit? For those that do not, explain why? **[6]**



(a) (b) (c)

A4. Discuss the validity of the following argument:

All graduates are educated.

Denis is a graduate.

Therefore. Denis is educated. **[3]**

SECTION B: Answer THREE questions in this section [60]

B5. (a) Draw the logic circuit L with inputs A, B, C and output Y which corresponds to each Boolean expression:

(i) $Y = ABC + A'C' + B'C'$ **[8]**

(ii) $Y = AB'C + ABC' + AB'C$ **[9]**

(b)Negate each of the following statements

(i) $\exists x \forall y, p(x, y)$ **[1]**

(ii) $\exists x \forall y, p(x, y)$ **[1]**

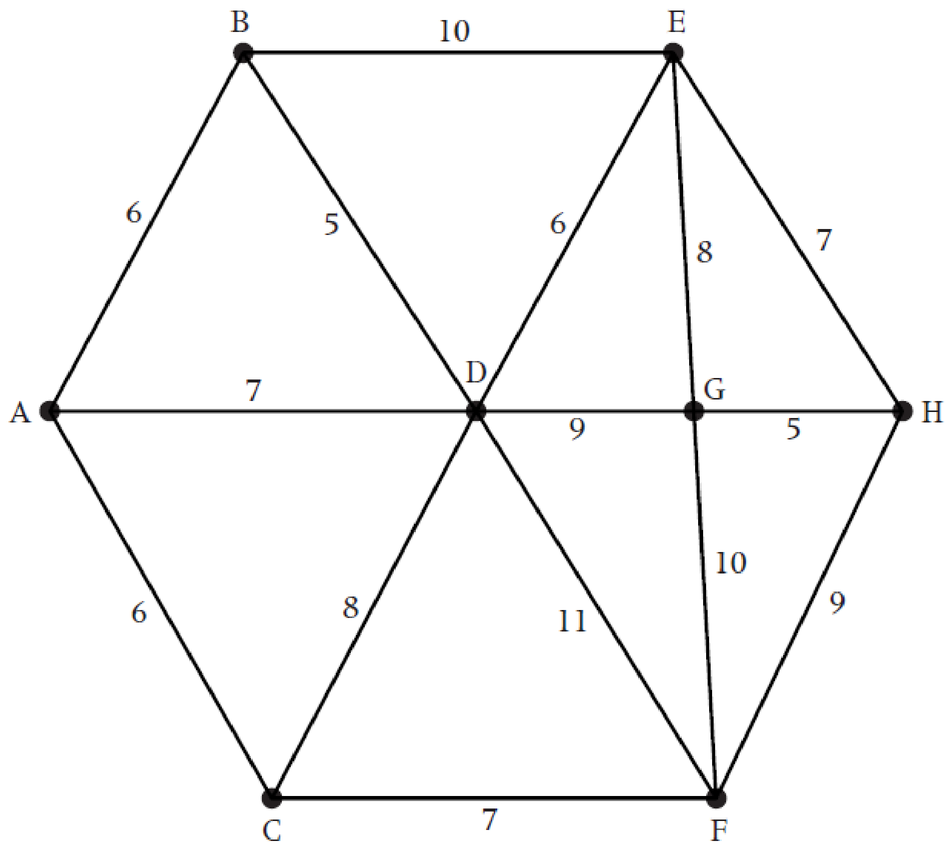
(iii) $\exists y \exists x \forall z, p(x, y, z)$ **[1]**

B6. (a) Show that $11^{n+2} + 12^{2n+1}$ is divisible by133. **[8]**

(b).Let $A = \{1,2\}$ construct that set $\rho(A) \times A$ where $\rho(A)$ is the power set of A . **[5]**

(c). Let U be a set of positive integers not exceeding1000. Then $|U| = 1000$. Find $|S|$ where S is the set of such integers which are not divisible by $3,5$ or 7. **[7]**

B7.(i) A Mutare City Council is responsible for maintaining the following network of roads. The number on each edge is the length of the road in kilometers.



(a) A Council worker has to inspect all the roads starting and finishing at A. Find the length of an optimal Chinese postman route. **[5]**

(b) A Supervisor based at A, also wish to inspect all the roads. However, the supervisor lives at H and wishes to start his route at A and finish it at H. Find the length of an optimal Chinese postman route for the supervisor. **[5]**

(ii) (a) Simplfy $\frac{(n+2)!}{n}$ **[4]**

(b). Prove $\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$ **[6]**

B8. (i) Draw the graph G corresponding to each adjacency matrix.

$$(a)\ A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

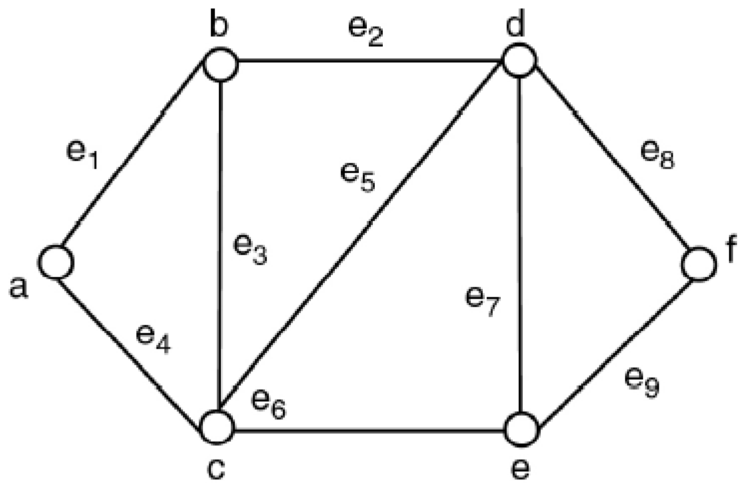
[5]

$$(b)\ A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

[5]

(ii) Prove the proposition that the sum of the first n positive integers is $\frac{1}{2}n(n+1)$ that is: $P(n): 1 + 2 + \cdots + n = \frac{1}{2}n(n+1)$ **[5]**

(iii) Construct a spanning tree of the following graph G



[5]

END OF EXAMINATION PAPER