MANICALAND STATE UNIVERSITY OF APPLIED SCIENCES

FACULTY OF ENGINEERING

DEPARTMENT OF MINING & MINERAL PROCESSING ENGINEERING
DEPARTMENT OF CHEMICAL & PROCESSING ENGINEERING

MODULE: ENGINEERING MATHEMATICS V

CODE: HGEN225

SESSIONAL EXAMINATIONS
OCTOBER 2021

DURATION: 3 HOURS

EXAMINER: DR W. GOVERE

INSTRUCTIONS

1. *Answer **All** in Section A*
2. *Answer **three** questions in Section B.*
3. *Start a new question on a fresh page*
4. *Total marks 100*

Additional material(s): *Non-programmable electronic scientific calculator, List of formulae.*

SECTION A: [40 MARKS]

*Answer **all** questions in this section*

A1.

(a) Prove that if B and C are both inverses of the matrix A, then $B = C$.

(b) Assuming that the sizes of the matrices are such that the indicated operations can be performed prove that

a. $A(B + C) = AB + AC$.

b. $A(BC) = (AB)C$

(c) Prove that

$$|A^{-1}| = \frac{1}{\det(A)}.$$

(d) Prove that if $ad - bc \neq 0$, then the reduced row echelon form of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

(e) Use the result in part (d) to prove that if $ad - bc \neq 0$, then the linear system

$$ax + by = k$$
$$cx + dy = l,$$

has exactly one solution.

[3, 3, 3, 3, 3, 3]

A2

(a) Find an integrating factor and solve the initial value problem $(e^{x+y} + ye^{y})dx + (xe^{y} - 1)dy = 0,\ y(0) = -1.$

(b) Find the general solution of the following differential equations:

i) $y' + y\tan x = \sin 2x$, and

ii) $(x^2 - y^2)dx + 3xydy = 0.$

[4, 4, 5]

A3

(a) Find the values of x and y in the equation
$x(1+i)^2 + y(2-i)^2 = 3 + 10i$ given further that $x, y \in \mathbb{R}$.

(b) The cubic equation $z^3 + Az^2 + Bz + 26 = 0$, $A, B \in \mathbb{R}$, has one of the roots as $1 + i$. Find the real root of the equation and hence determine values for A and B.

[4, 5]

SECTION B: [60 MARKS]

Answer any <u>three</u> questions

B4

(a) Prove that if R is the reduced row echelon form of an $n \times n$ matrix A, then either R has a row of zeros or R is the identiry matrix I_n.

(b) Use row reduction to show that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (b-a)(c-a)(c-b).$$

(c) Solve the following systems of nonlinear equations for x, y and z:

$$x^2 + y^2 + z^2 = 6$$
$$x^2 - y^2 + 2z^2 = 2$$
$$2x^2 + y^2 - z^2 = 3$$

(d) Find the value of a & b for which the following systems of equations

$$x - y + 2z = 4$$
$$3x - 2y + 9z = 14$$
$$2x - 4y + az = b$$

have

i. no solution.
ii. unique solution.
iii. infinitely many solutions.

[4, 4, 5, 3, 2, 2]

B5

(a) Prove that if $f(t)$ has the transform $F(s)$ (where $s > k$ for some k), then $e^{at}f(t)$ has the transform $F(s-a)$ (where $s - a > k$).

(b) *The transforms of the first and second derivatives of f(t) satisfy*

(1). $\mathcal{L}(f') = s\mathcal{L}(f) - f(0)$

(2). $\mathcal{L}(f'') = s^2\mathcal{L}(f) - sf(0) - f'(0)$

(1) *Holds if $f(t)$ is continuous for all $t \geq 0$ and satisfies the growth restriction and $f'(t)$ is piecewise continuous on every finite interval on*

the semi-axis $t \geq 0$. Similarly, (2) holds if f and f' are continuous for all $t \geq 0$ and satisfy the growth restriction and f'' is piecewise continuous on every finite interval on the semi-axis $t \geq 0$.

Prove the above theorem.

(c) Find the function $f(x)$ such that

$$\mathcal{L}\{f(x)\} = \frac{s}{(s-2)(s+1)}$$

(d) Solve the equation

$$y'' + 3y' + 2y = cos\, 2x,$$

given that $y(0) = 1$ and $y'(0) = 0$.

[4, 4, 4, 8]

B6

(a) Water is heated to the boiling point temperature 100℃. It is then removed from heat and kept in a room which is at constant temperature of 60℃. After 3 minutes, the temperature of the water is 90℃.

i. Find the temperature of the water after 6 minutes.

ii. When will the temperature of water be 75℃ and 61℃?

(b) Prove that for a $y'' + p(x)y' + q(x)y = 0$, any linear combination of two solutions on an open interval I is again a solution of $y'' + p(x)y' + q(x)y = 0$ on I. In particular prove that, for such an equation, sums and constant multiples of solutions are again solutions.

(c) Solve the IVP:

$$\cos(x)y' + \sin(x)y = 2\cos^2(x)\sin(x) - 1,$$

given that

$$y\left(\frac{\pi}{4}\right) = 3\sqrt{2}, \qquad 0 \leq x \leq \frac{\pi}{2}$$

.

(d) Given that $y_1 = e^x$ is a solution of $y'' - y = 0$ on the interval $(-\infty, \infty)$, use reduction of order to find a second solution y_2.

(e) Using the method of undetermined coefficients find the general solution of $y'' + 4y' - 2y = 2x^2 - 3x + 6$.

[2, 3, 3, 4, 4, 4]

B7

a) Assume that the level of a certain hormone in the blood of a patient varies with time. Suppose that the time rate of change is the difference between a sinusoidal input of a 24-hour period from the thyroid gland and a continuous removal rate proportional to the level present. Set up a model for the hormone level in the blood and find its general solution. Find the particular solution satisfying a suitable initial condition.

b) Determine the convergence set for

$$\sum_{n=0}^{\infty} \frac{2^{-n}}{n+1}(x-1)^n$$

c) Find a power series expansion about $x = 0$ for a general solution to the given differential equations:

i. $y'' + x^2 y = 0.$

ii. $(1 - x^2)y'' - 2xy' + 2y = 0.$

(a) Find and classify the singular points of

$$(x^2 - 4)^2 y'' + 3(x - 2)y' + 5y = 0.$$

[4, 4, 4, 4, 4]

END OF QUESTION PAPER