MANICALAND STATE UNIVERSITY OF APPLIED SCIENCES

FACULTY OF ENGINEERING, SCIENCE AND TECHNOLOGY

DEPARTMENT: CHEMICAL AND PROCESSING ENGINEERING

MODULE: FLUID FLOW II

CODE: CHEP312

SESSIONAL EXAMINATIONS
JUNE 2023

DURATION: 3 HOURS

EXAMINER: MR D NYADENGA

INSTRUCTIONS

1. *Answer **any four** questions.*
2. *Start a new question on a fresh page.*
3. *Total marks 100.*
4. *Use of scientific calculator is permitted.*

Additional material(s):None

QUESTION 1

a) Define the following terms:

i. Fluid [1]

ii. Streamline [1]

iii. Pathline [1]

b) Distinguish between the *Lagrangian* and *Eulerian* approaches to flow phenomena. [2]

c) A steady, incompressible, two-dimensional velocity field is given by the following components in the *x*-*y* plane:

$$\text{v} = (1.85 + 2.33x + 0.656y)\boldsymbol{i} + (0.754 - 2.18x - 2.33y)\boldsymbol{j}$$

Calculate the acceleration field at a point where x = -1 and y = 2 m (find acceleration components a_x and a_y. [12]

d) By performing a mass balance on a volume element $(\Delta x)(\Delta y)(\Delta z)$, show that for a fluid of constant density, the equation of continuity reduces to:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \text{ or } \nabla \cdot \text{v} = 0$$ [8]

QUESTION 2

An incompressible Newtonian fluid flows steadily in a narrow gap between two very long, wide, parallel plates. The gap between the plates is h. The top plate moves at speed U in the x-direction whilst the bottom plate is stationary. Gravity acts in the negative z-direction (into the page). A constant negative pressure gradient applied in the x-direction necessitates the flow of the fluid.

a) State any assumptions that simplify the problem. [4]

b) Using the Continuity equation and Navier-Stokes equation, show that:

$$-\left(-\frac{\partial P}{\partial x}\right) + \mu \frac{\partial^2 v_x}{\partial y^2} = 0$$ [12]

c) By applying suitable boundary conditions, determine the velocity profile of the flow. [9]

QUESTION 3

a) Under what conditions does Navier-Stokes equation reduce to:

i. hydrostatic pressure equation? [2]

ii. Bernoulli's equation? [4]

b) By applying the conditions stated in part a) i. on all components of the Navier-Stokes equation, show that when gravity is acting in the positive z-direction (downwards), the pressure difference between points 1 and 2 (point 2 is below point 1) is given by:

$$P_2 - P_1 = \rho g(z_2 - z_1)$$ [12]

c) A 3 m high large tank is initially filled with water. The tank water surface is open to the atmosphere, and a sharp-edged 10 cm diameter orifice at the bottom drains to the atmosphere through a horizontal 80 m long pipe. This is shown in Figure 1.

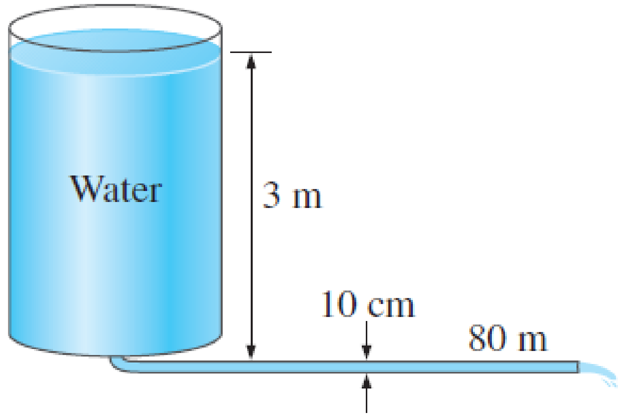

Figure 1

If the total irreversible head loss of the system is determined to be 1.5 m, determine the initial velocity of water from the tank. [7]

QUESTION 4

a) State ***two*** conditions that result in the existence of a stream function. [2]

b) For a certain incompressible flow field, the velocity is given by:

$$\mathrm{v} = (x^2 - y^2)\boldsymbol{i} - 2xy\boldsymbol{j}$$

Determine the following:

i. Whether the continuity equation is satisfied. [5]

ii. The stream function [9]

iii. The velocity potential [9]

QUESTION 5

a) Define the term *boundary layer*. [1]

b) With the aid of a diagram, describe the boundary layer phenomena when a fluid with uniform velocity flows past a thin flat plate. [10]

c) Air at 27 °C flows at a velocity of 5 m/s over a smooth flat plate of length 15.2 m.

i. Calculate the distance from the leading edge when the flow transitions from laminar to turbulent. [4]

ii. Determine the boundary layer thickness at a point that is 1.5 m from the leading edge. [4]

iii. If it is water at the same temperature and flowing at the same velocity, determine the boundary layer thickness at a point that is 1.5 m from the leading edge. [6]

Note: Use $Re_{x,transition} = 3 \times 10^6$
The physical properties of air and water are attached in the question paper.

END OF EXAMINATION
FORMULAE SHEET

Navier-Stokes equations

x-component: $$-\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 \mathrm{v}_x}{\partial x^2} + \frac{\partial^2 \mathrm{v}_x}{\partial y^2} + \frac{\partial^2 \mathrm{v}_x}{\partial z^2}\right) + \rho g_x = \rho\frac{D\mathrm{v}_x}{Dt}$$

y component: $$-\frac{\partial P}{\partial y} + \mu\left(\frac{\partial^2 \mathrm{v}_y}{\partial x^2} + \frac{\partial^2 \mathrm{v}_y}{\partial y^2} + \frac{\partial^2 \mathrm{v}_y}{\partial z^2}\right) + \rho g_y = \rho\frac{D\mathrm{v}_y}{Dt}$$

z-component: $$-\frac{\partial P}{\partial z} + \mu\left(\frac{\partial^2 \mathrm{v}_z}{\partial x^2} + \frac{\partial^2 \mathrm{v}_z}{\partial y^2} + \frac{\partial^2 \mathrm{v}_z}{\partial z^2}\right) + \rho g_z = \rho\frac{D\mathrm{v}_z}{Dt}$$

Substantial derivative: $$\frac{D()}{Dt} = \frac{\partial()}{\partial t} + \mathrm{v}_x\frac{\partial()}{\partial x} + \mathrm{v}_y\frac{\partial()}{\partial y} + \mathrm{v}_z\frac{\partial()}{\partial z}$$

Boundary layer thickness equations

Laminar flow: $$\frac{\delta}{x} = \frac{4.91}{\sqrt{Re_x}}$$

Turbulent flow: $$\frac{\delta}{x} = \frac{0.38}{(Re_x)^{\frac{1}{5}}}$$

Physical properties of air

T (K)	ρ (kg/m^3)	$c_p \times 10^{-3}$ (J/kg × K)	$\mu \times 10^5$ (Pa × s)	$\nu \times 10^5$ (m^2/s)	$k \times 10^2$ (W/m × K)	$\alpha \times 10^5$ (m^2/s)	Pr	$g\beta\rho^2/\mu^2$ (1/K · m^3)
				Air				
250	1.4133	1.0054	1.5991	1.1315	2.2269	1.5672	0.722	4.638×10^8
260	1.3587	1.0054	1.6503	1.2146	2.3080	1.6896	0.719	2.573
280	1.2614	1.0057	1.7503	1.3876	2.4671	1.9448	0.713	1.815
300	1.1769	1.0063	1.8464	1.5689	2.6240	2.2156	0.708	1.327
320	1.1032	1.0073	1.9391	1.7577	2.7785	2.5003	0.703	0.9942
340	1.0382	1.0085	2.0300	1.9553	2.9282	2.7967	0.699	0.7502
360	0.9805	1.0100	2.1175	2.1596	3.0779	3.1080	0.695	0.5828
400	0.8822	1.0142	2.2857	2.5909	3.3651	3.7610	0.689	0.3656
440	0.8021	1.0197	2.4453	3.0486	3.6427	4.4537	0.684	0.2394
480	0.7351	1.0263	2.5963	3.5319	3.9107	5.1836	0.681	0.1627
520	0.6786	1.0339	2.7422	4.0410	4.1690	5.9421	0.680	0.1156
580	0.6084	1.0468	2.9515	4.8512	4.5407	7.1297	0.680	7.193×10^6
700	0.5040	1.0751	3.3325	6.6121	5.2360	9.6632	0.684	3.210
800	0.4411	1.0988	3.6242	8.2163	5.7743	11.9136	0.689	1.804
1000	0.3529	1.1421	4.1527	11.1767	6.7544	16.7583	0.702	0.803

Physical properties of water

T (K)	ρ (kg/m^3)	c_p (J/kg × K)	$\mu \times 10^6$ (Pa × s)	$\nu \times 10^6$ (m^2/s)	k (W/m × K)	$\alpha \times 10^6$ (m^2/s)	Pr	$g\beta\rho^2/\mu^2 \times 10^{-9}$ (1/K · m^3)
				Water				
273	999.3	4226	1794	1.795	0.558	0.132	13.6	
293	998.2	4182	993	0.995	0.597	0.143	6.96	2.035
313	992.2	4175	658	0.663	0.633	0.153	4.33	8.833
333	983.2	4181	472	0.480	0.658	0.160	3.00	22.75
353	971.8	4194	352	0.362	0.673	0.165	2.57	46.68
373	958.4	4211	278	0.290	0.682	0.169	1.72	85.09
473	862.8	4501	139	0.161	0.665	0.171	0.94	517.2
573	712.5	5694	92.2	0.129	0.564	0.139	0.93	1766.0