MANICALAND STATE UNIVERSITY OF APPLIED SCIENCES

FACULTY OF ENGINEERING

DEPARTMENT OF MINING & MINERAL PROCESSING ENGINEERING
DEPARTMENT OF CHEMICAL & PROCESSING ENGINEERING
DEPARTMENT OF METALLUGICAL ENGINEERING

MODULE: ENGINEERING MATHEMATICS IV

CODE: ENGT 224

SESSIONAL EXAMINATIONS
April 2024

DURATION: 3 HOURS

EXAMINER: J. MANYEMBA

INSTRUCTIONS

1. *Answer **All questions** in Section A*
2. *Answer **three** questions in Section B.*
3. *Start a new question on a fresh page*
4. *Total marks 100*

***Additional material(s):** Non-programmable electronic scientific calculator.*

Statistical tables

SECTION A [40 marks]

Answer **ALL** Questions being careful to number them A1 to A3.

A1. When do we apply the following procedures:

(a) χ^2-test for association. [2]

(b) regression analysis. [2]

A2. Let $y_i = \beta_0 + \beta_1 x_i + \epsilon_i,\ i = 1, ..., n$, be a linear regression model where β_0 and β_1 are parameters and ϵ_i is the error term,

(a) Derive the least squares estimators for β_0 and β_1. [7]

(b) Show that the least squares estimators for β_0 and β_1 are unbiased. [6]

A3. The use of bank cards as a source of consumer credits has become increasingly prevalent. A recent study has attempted to profile holders of ATM, VISA or both cards.The data is shown in Table 1.

Table 1:

	Credit Card Held		
Sex	ATM	Visa Only	Both Cards
Male	128	66	137
Female	295	165	287

From the data shown in the accompanying Table, would you conclude that there is a relationship between gender and the credit card held? Use $\alpha = 0.05$. [12]

A4. Table 2 gives observations on mathematics achievement test score (x_i) and calculus grades (y_i) for ten independently selected college freshmen.

Table 2:

x_i	39	43	21	64	57	47	28	75	34	52
y_i	65	78	52	82	92	89	73	98	56	75

(a) Draw a scatterplot for these data and comment. [4]

(b) Calculate Spearman's correlation coefficient and comment. [4]

(c) Test at the $\alpha = 0.01$ level of significance if $\rho_S = 0$. [3]

SECTION B [60 marks]

Answer any **THREE** Questions being careful to number them B4 to B7.

B5. Consider the following data set in Table 3 , where Y is the dependent variable and X_1 and X_2 are the regressors.

Table 3:

Y	52	38	36	29	43
X_1	246	241	288	191	248
X_2	18	14	13	15	21

Suppose the data can be described by model $Y_i = \beta_0 + \beta_1 X_1 i + \beta_2 X_2 i + \epsilon_i$ where $\epsilon_i \sim N(0, \sigma^2)$ and $Cov(e_i, e_j) = 0$ if $i \neq j$.

(a) Express the above model in matrix form, defining all the terms. [2]

(b) Find the least squares estimate of β given that

$$[\mathbf{X'X}]^{-1} = \begin{bmatrix} 20.77245 & -0.05535 & -0.44032 \\ -0.05535 & 0.00021 & 0.00024 \\ -0.44032 & 0.00024 & 0.02363 \end{bmatrix}$$

[9]

(c) Given that $SS_{regression} = 160.5484$, construct the ANOVA table and test for the significance of the regression line using $\alpha = 0.05$. [9]

B6. A scientist feels that his new brand of rat poison is superior to two other brands. Each product is tested on five rats (each) and the survival times (in minutes) are shown in Table 4 ,

Table 4:

Brand A (Scientist's Brand)	Brand B	Brand C
3.95	4.01	4.30
3.87	4.44	5.70
3.25	4.90	5.00
3.12	4.75	5.23
3.75	4.28	4.75

(a) State the one-way analysis of variance model and the assumptions required for its validity. [5]

(b) Estimate the parameters μ, the constant and τ_1, τ_2, τ_3, the treatment effects. [5]

(c) Perform a one-way analysis of variance on the data and test the appropriate hypothesis. [10]

B7. The following data in Table 5 are from a study of how climatic variables affect the incidence of rust disease on winter wheat. Cold days (X) are a measure of the severity of winter over a 12 year period.

Table 5:

Cold Days (X)	570	500	460	700	570	630	400	480	870	520	480	460
Disease index (Y)	4.0	5.5	5.5	3.0	3.0	4.0	7.0	7.0	1.0	4.0	4.0	6.0

(a) Fit the regression model to this data using the method of least squares. [5]

(b) Construct the ANOVA table and test for significance of the regression line. Use $\alpha = 0.05$. [10]

(c) Derive the 95% confidence interval of the intercept. [5]

B8. (a) State four assumptions underlying the Durbin Watson test and the ways to validate them. [8]

(b) Data for soft drinks concentrate sales were collected annually. A regression model was fitted with expenditure as a predictor variable. Part of the results of the analysis are given below:

$$n = 20, \qquad \sum_{t=1}^{20} e_t^2 = 7587.9154, \qquad \sum_{t=2}^{20} (e_t - e_{t-1})^2 = 8195.2065$$

Test for positive autocorrelation using the Durbin-Watson test. Use $\alpha = 0.05$ [12]

END OF EXAMINATION PAPER

TABLE A.6 Durbin-Watson Test Bounds

Level of Significance $\alpha = .05$.

	$p-1=1$		$p-1=2$		$p-1=3$		$p-1=4$		$p-1=5$	
n	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
15	1.08	1.36	0.95	1.54	0.82	1.75	0.69	1.97	0.56	2.21
16	1.10	1.37	0.98	1.54	0.86	1.73	0.74	1.93	0.62	2.15
17	1.13	1.38	1.02	1.54	0.90	1.71	0.78	1.90	0.67	2.10
18	1.16	1.39	1.05	1.53	0.93	1.69	0.82	1.87	0.71	2.06
19	1.18	1.40	1.08	1.53	0.97	1.68	0.86	1.85	0.75	2.02
20	1.20	1.41	1.10	1.54	1.00	1.68	0.90	1.83	0.79	1.99
21	1.22	1.42	1.13	1.54	1.03	1.67	0.93	1.81	0.83	1.96
22	1.24	1.43	1.15	1.54	1.05	1.66	0.96	1.80	0.86	1.94
23	1.26	1.44	1.17	1.54	1.08	1.66	0.99	1.79	0.90	1.92
24	1.27	1.45	1.19	1.55	1.10	1.66	1.01	1.78	0.93	1.90
25	1.29	1.45	1.21	1.55	1.12	1.66	1.04	1.77	0.95	1.89
26	1.30	1.46	1.22	1.55	1.14	1.65	1.06	1.76	0.98	1.88
27	1.32	1.47	1.24	1.56	1.16	1.65	1.08	1.76	1.01	1.86
28	1.33	1.48	1.26	1.56	1.18	1.65	1.10	1.75	1.03	1.85
29	1.34	1.48	1.27	1.56	1.20	1.65	1.12	1.74	1.05	1.84
30	1.35	1.49	1.28	1.57	1.21	1.65	1.14	1.74	1.07	1.83
31	1.36	1.50	1.30	1.57	1.23	1.65	1.16	1.74	1.09	1.83
32	1.37	1.50	1.31	1.57	1.24	1.65	1.18	1.73	1.11	1.82
33	1.38	1.51	1.32	1.58	1.26	1.65	1.19	1.73	1.13	1.81
34	1.39	1.51	1.33	1.58	1.27	1.65	1.21	1.73	1.15	1.81
35	1.40	1.52	1.34	1.58	1.28	1.65	1.22	1.73	1.16	1.80
36	1.41	1.52	1.35	1.59	1.29	1.65	1.24	1.73	1.18	1.80
37	1.42	1.53	1.36	1.59	1.31	1.66	1.25	1.72	1.19	1.80
38	1.43	1.54	1.37	1.59	1.32	1.66	1.26	1.72	1.21	1.79
39	1.43	1.54	1.38	1.60	1.33	1.66	1.27	1.72	1.22	1.79
40	1.44	1.54	1.39	1.60	1.34	1.66	1.29	1.72	1.23	1.79
45	1.48	1.57	1.43	1.62	1.38	1.67	1.34	1.72	1.29	1.78
50	1.50	1.59	1.46	1.63	1.42	1.67	1.38	1.72	1.34	1.77
55	1.53	1.60	1.49	1.64	1.45	1.68	1.41	1.72	1.38	1.77
60	1.55	1.62	1.51	1.65	1.48	1.69	1.44	1.73	1.41	1.77
65	1.57	1.63	1.54	1.66	1.50	1.70	1.47	1.73	1.44	1.77
70	1.58	1.64	1.55	1.67	1.52	1.70	1.49	1.74	1.46	1.77
75	1.60	1.65	1.57	1.68	1.54	1.71	1.51	1.74	1.49	1.77
80	1.61	1.66	1.59	1.69	1.56	1.72	1.53	1.74	1.51	1.77
85	1.62	1.67	1.60	1.70	1.57	1.72	1.55	1.75	1.52	1.77
90	1.63	1.68	1.61	1.70	1.59	1.73	1.57	1.75	1.54	1.78
95	1.64	1.69	1.62	1.71	1.60	1.73	1.58	1.75	1.56	1.78
100	1.65	1.69	1.63	1.72	1.61	1.74	1.59	1.76	1.57	1.78